Proceedings of the 8th International Conference on Applied Informatics
Eger, Hungary, January 27–30, 2010. Vol. 2. pp. 263–274.

Game Theoretic Approach of Product Distribution by Vehicles

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Abstract

This paper describes our algorithm for determining how to partition the set of destination points among a set of suppliers in a product distribution situation for reaching the maximal profit. We present both the theory of the initial distribution and the redistribution of destinations in case of an unhoped-for event (e.g. accident, sudden unexpected order, etc.). In this work we use a game theoretic approach [1], which completes well the recent VRP (Vehicle Routing Problem) solutions. We analyze the cases of both cooperative and competitive (non-cooperative) suppliers.

We show that our solution for the non-cooperative case is a Nash-equilibrium of this "game", while in cooperative case we give the Nash-product of the problem using the players' parameters. We think that especially our cooperative game theoretic approach is a new and promising idea in this topic, that's why the cooperative part of this issue is highlighted more.

After introducing the theoretic background we present a concrete real-life problem of product distribution and use our results for solving it.

Keywords: Game theory, Nash-product, logistics, adaptation, cooperation

1. Introduction

Logistic tasks surround us in our global world: supermarkets have to get the products from factories, public vehicles have to transport people, trucks have to deliver different goods for people. The logistic market is huge and naturally its suppliers intend to maximize their own profit. It means that each company wants to snatch as big part of the market as he can and seeks after minimal expenses. It results that the members of the same company cooperate while they compete with the members of other companies. Their profit and expense primarily depends on the number of clients they serve, their road and the price they determine for their goods.

It seems to be mainly a routing problem. For the initial distribution of the destination points it is really a routing problem with several parameters. However, when the tracks are on the move and an unexpected event (e.g. accident, extra order) occurs, the initial distribution may change for maximizing the profit.

Our days' solutions for dynamical routing problem are in one's infancy, however for static routing there are several services. Some of them are online services, e.g. ViaMichelin [2], while others can be used as real-time navigation systems (iGo [3], myGuide [4], etc.). Their update happens rarely by changing the map, that's why they can't take into account any changes of the roads and traffic. Their usage is widely spread and several complementary services are built onto them, however their biggest drawback is the static information they provide. In 1996 a system was tested for solving the traffic problems of Tokyo [5]: Centrally Determined Route Guidance (CDRG) needed built-in infrared transceivers in the cars and some centrally managed communication stations in the city. This system was able to provide the route-planning software with real-time traffic information. There are other dynamic route guidance solutions, which use radio-communication. They use Traffic Message Channel (TMC) in the FM radio band to carry weather and traffic information. It can be used well in real-time navigation. Unfortunately this method needs huge investments on supplier side to build up the service's infrastructure. It puts the lid on the method's spread.

Discussing not only the individual route-planning, but the fleet management methods of our days, most common methods in the practice use human dispatcher for organizing the fleet (e.g. taxi companies, police). Although the problem of computer based initial routing is a rich research area nowadays. It has the name VRP (Vehicle Routing Problem). There are several variants of it: VRP with determined time window at the customers (VRPTW – Vehicle Routing Problem with Time Window) [16], VRP where vehicles have fixed capacity (just like in a real life problem) (CVRP – Capacitated Vehicle Routing Problem) [19], etc. [15]

Contrarily there is another approach of the problem: real-time decisions for the route guidance are made individually by the members of the fleet based on the information they got from their environment. Rich research area is the topic of Advanced Fleet Management Systems (AFSM). Some of them use experience and statistical results while others work with stochastic networks [6, 7, 8].

For the above mentioned problem – a fleet member faces an unexpected change in his environment which influences his road /and perhaps other members' road too/ - we have not already find any solution in the literature.

Our paper covers route planning problems highlighting the cooperation between agents of a company, however we advert briefly to the competition with the competitors' agents too. First we collect the parameters which play important role in the final outcome (the profit). Based on these parameters we determine the players' profit functions.

In Section 2 we present our solution for the initial distribution of the destination points and compare it with the existing methods. Section 3 shows our method of the adaptation for the changed circumstances while completing a transmission task.

We deal both with cooperative and non-cooperative cases. After that we show a case study using our game theoretic approach in Section 4. Finally we conclude our work.

2. How to supply several customers by more agents in an efficient way?

Suppose that there are n customers (n>>1) and m suppliers (agents) (m>=1). All the customers have to be served by the suppliers with the same kind of goods. Each customer has non zero amount (A_i, where $0 < i <= n$) of demand. All the needs have to be satisfied. In this solution we don't deal with the capacity of the agents, only the travel time: suppose that distributors have infinite capacity. We have to answer two questions:

1. How to distribute the customers (destination points) among the agents?
2. How to route an agent in his destination point set?

For these questions researches in the area of VRP have already given answer. The fact is that this problem is NP-complete, that's why all the solutions apply heuristic for finding a satisfying answer. Several methods have been applied for solving the problem, some of them apply the results of linear computing, some of them use fuzzy logics [17, 18], while others are based on genetic algorithm [14, 19, 20, 21]. We try to give another approach for solving VRP, its effectiveness is showed by using game theory.

This Section shows our approach as answer for the questions above.

2.1. Initial destination point distribution

In the simplest case m=1. In this case all the n customers have to be visited by the only one agent. When there is more than one agent, we have to separate the solutions into cooperative and non-cooperative cases.

When we consider cooperative agents, there is one common goal: to satisfy all the customers' needs with minimal charge. The charge origins from the cost of the travel and is commensurable to the length of the path the agents have to tour. When we suppose that the travel time is related with the length of the road in a linear way, our task is to find a distribution of destination points which can be served in a minimal time. (Here time means the sum of all agents' individual time values.)

For solving this task our algorithm is as follows:

1. For each agent do: set path length to zero ($l_j = 0$). At this phase an agent's path means only one point: the agent's originating point.

2. For each destination point (n) do:
    For each agent (m) do:
        Let's determine the distance between the destination point and the agent's path ($d_{i,j}$, where $0 < i <= n$ and $0 < j <= m$) [9]

3. Select the minimal $d_{i,j}$. It determines that destination point i will be served by agent j. Modify agent j's path and increase his path length: $l_j = l_j + d_{i,j}$. Remove destination point i from the set of destination points.
4. If the set of destination points is not empty, GOTO 2, else quit.

This algorithm ensures a nearly minimal cost of transportation. In extreme cases all the destination points can be served by only one agent. It means that the common goal can be satisfied (the company could be pleased), however the income and work is not divided definitely equally between the agents.

When the agents don't belong to the same company and they compete with each other, the solution differs. In this non-cooperative case there is not a common goal. Each agent intends to maximize her own income in a selfish way and is thoughtless of the others. The solution of this problem is a Nash-equilibrium. Nash equilibrium is a strategy profile of a non-cooperative game, where no player has interest in changing his own strategy unilaterally [10, 11]. It means that we have to find those states of this problem, where there is no chance for increasing the agents' profit. There are two parameters of each agent-destination point pair: the price of the goods the agent carries to the customer in the destination point ($P_{i,j}$) and the cost of the transport ($TC_{i,j}$), where i comes from the set of destination points and j from the set of agents. If we consider only one agent-destination point pair, the transmission is rewarding for an agent when $TC_{i,j} < P_{i,j}$. A customer - suppose that he is a rational decision maker - chooses the agent with the lowest prize. In the competition the agent is the winner, who can offer goods with the lowest price. We have to answer these questions: Which agent wins? What price should he set?

Agent j wins the competition for destination point i when he can set his price to such a value, that there is no other agent whom this price is worth. It means that agent j's path is the nearest path to destination point i. (Otherwise there is another agent who can decrease his price underbidding agent j.) Answering the two questions, that agent wins who has the nearest path to the destination point. He should set the price to be just under the estimated cost of the second "cheapest" agent.

Based on the above mentioned theory our algorithm for creating the initial non-cooperative distribution is the same as in the cooperative case. The distribution is based on the distance between the suppliers' path and the destination points.

2.2. Routing problem

Let's start again with the simplest case: there is only one agent ($m = 1$) who has to serve all the customers. (It is equal to the case when all destination points have the same nearest agent path.) The problem is: we have to find the agent with the shortest path which visits all the destination points. It is exactly the Traveling Salesman Problem. Although TSP is NP-complete problem, because of its popular research there are several algorithms for solving TSP with more and more destination points. [12, 13]

When m > 1 each agent's destination point set can be handled separately. It means that for answering the question "How to route an agent in his destination point set?" we can use the results of TSP-related researches.

3. Adaptive handle of changes during transportation

Assume that there is an initial partitioning of destination point set and different partitions are assigned to different suppliers. In this Section we will examine if it is worth to modify the partitions in case of change in the circumstances (accident, extra order, etc.) and when it is, how to do that.

3.1. The parameters of the problem

First of all we collect the parameters which are important in solving the above mentioned problem. Let's consider now only the simplest case: there is only one customer, who may be served by not his original supplier. We have to decide weather to assign this customer to other agent. For doing this task we have to separate the suppliers into two distinct sets. The first set has only one member, he is the agent whose initial path is directly modified because of the changes in the circumstances. The second set of suppliers contains all the other agents. The parameters of the first set - one simple agent - we deal are:

- OE (own expense): it would be his own transport expenses if he carries the goods to the actual customer. When it is impossible to serve the customer (e.g. the track broke down), this value is set to be infinite.
- TC: it is the charge he has to pay to the agent who takes over his work.

The other agents are members of the second set. All of them have the following parameters:

- E_i: the extra transport expense of agent i when he serves the problematic customer.
- P: the price of the goods. He gets it from the customer and is determined by the customer's original supplier.
- TC: it is the income from the original supplier as a charge of the work. It is equal to the previously introduced parameter "TC".

Based on these parameters the profit of all agents can be determined. The profit of the agent in the first set - let's call him agent A - is:

$$\mathrm{OE} - \mathrm{TC}.$$

It can be seen that we handled his possible transport expenses as his income. The reason for it is that if other agent takes over his work, agent A hasn't to pay this price of the transport. He only has to pay for the other agent's work TC. P doesn't influence his budget, because goods remain at agent A.

Agents in the second set have profit

$$\mathrm{P} + \mathrm{TC} - \mathrm{E}_i.$$

They may got income P from the customer, TC from agent A and their only cost is their extra transport expense E_i.

3.2. How to change the initial partitioning of destination points?

Previously we gave an initial partitioning of customers and introduced some parameters of the suppliers. For the sake of simplicity first we suppose that there is only one customer whose serving is endangered because of e.g. an accident. The first question we have to answer: Weather the customer's original supplier (agent A) should serve the customer or shall he pass this transfer to other agent?

For answering this question let's examine the profits of the agents. Agent A will pass a customer to another agent only if it is worth for him. It happens when his profit value is bigger than zero:

$$\mathrm{OE} - \mathrm{TC} > 0.$$

Examining this question from the other side: Is there other agent whom it is worth to take over agent A's task? When an agent's profit would be bigger than zero and he is a rational decision maker, he would take over agent A's customer. It means that agent A can pass his task to other agent if there is agent j, whose parameters satisfy:

$$\mathrm{P} + \mathrm{TC} - \mathrm{E}_j > 0.$$

As P is a constant value (determined previously by agent A) and transport expenses (OE and E_i-s) depend on the initially determined paths (and can be calculated based on the initial paths and the actual GPS positions of the agents), there is only one variable – TC – which may influence whether an agent would get positive profit value. (For the sake of correctness an agent who would take over the task may decrease the value of P if it also worth for him, however now we don't care this case.)

First we have to calculate OE and all the E_i values. They depend on the distance between the destination point and the agents' path. After that if there is a value TC for which OE – TC > 0 and there is agent i for which P + TC – E_i > 0, agent A will pass his task to another agent. Naturally more agents may have positive profit values (P + TC – E_j > 0 and P + TC – E_k > 0, where $j \neq k$). In this case a new question arises: Which agent to pass the task to?

As agent with the maximal profit may decrease the value of the desired charge (TC) the most, the answer is that agent with the maximal profit value (agent with the minimal transfer cost) will won this game:

$$\min_i E_i.$$

The final question we have to answer is: How to determine the charge of the passed work (TC)?

Expressing from the profit equations (supposing that agent A passes the work to another supplier):

$$E_i - P < TC < OE.$$

At this point we have to separate the solution into two parts. First assume that agents belong to the same company and they cooperate. In this case the main goal is to minimize the overall expenses. It means that they intend to maximize the overall profit. Using the profit functions of the agents we may write a Nash-product of this cooperative game:

$$\prod_i (P + TC - E_i) * (OE - TC)$$

The Nash-equilibrium of a cooperative game can be determined by maximizing the Nash-product. In this case there is no means to speak about TC as all the agents work for he same company. It means that this product reaches its maximum value if and only if E_i is minimal. It means that agent with the nearest path has to serve the problematic destination point.

In the case when the agents (or groups of agents) compete with each other, the game is a typical non-cooperative game. For finding its Nash-equilibrium we have to find a strategy profile from which no agents intends to differ unilaterally. It means that

$$TC = OE - \varepsilon,\ \varepsilon > 0,$$

else there is at least one agent who may want to differ from the determined value of TC.

There may be a situation when the pass of one destination point infers the pass of other destination points too. For these kinds of situations the same computation has to be done. (E.g. in case of 2 problematic destination points and N agents there are N^2 possible outcomes.)

4. Case study: gas bottle delivery

In this section we raise a real-life problem of product distribution and attempt to apply our method to find the solution of the problem.

4.1. Description of the problem

For the sake of simplicity we feature only a small number of agents and customers in our example. In this small part of the bigger real-life situation there are 2 agents and 10 customers. Agents' repositories are in two Hungarian cities: one of them is in Gógánfa (agent 1), while the other is in Székesfehérvár (agent 2). Suppose that there is an initial destination point separation, which assures the quickest (and cheapest) delivery. Figure 1 shows it. Arrows sing the direction of their path. We can see that each agent starts from his repository, has to serve 5 customers and has to arrive at the repository. Both agents' path is a circle.

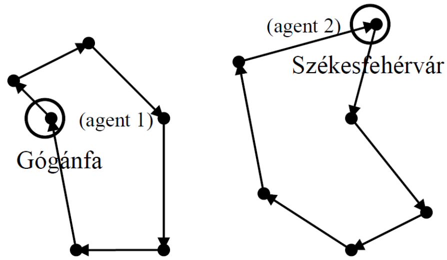

Figure 1: Case study: Initial destination point separation

Suppose that agent 2 (whose repository is in Székesfehérvár) is on his road and has served all his destination points excepting the latest one (point B in Figure 2). At this time he has just leaved the previous customer (destination point D in Figure 2), and has to face the fact that the road from D to B is closed. This problem is presented in Figure 2. Actual positions of the agents are signed with flags. The supplier from Gógánfa (agent 1) will just arrive at her destination point A.

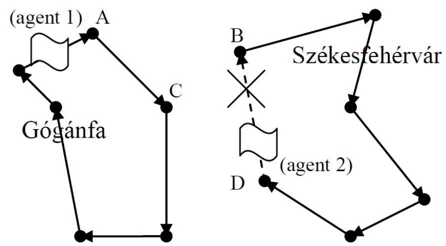

Figure 2: Case study: agent 2 faces a traffic problem

Agent 2 has to decide what do to. In order to getting to know his possibilities we've illustrated the roads and distances which play important role in finding the solution of the problem in Figure 3.

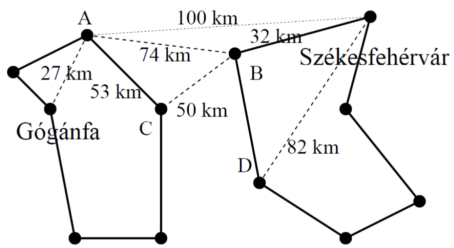

Figure 3: Case study: most important roads and distances

If agent 2 insists on serving customer in point B, the shortest possibility for him is to go back in Székesfehérvár and serve destination point B from there (supposing that distance between points D and C is high enough – in our case bigger than 64 km, or there is no direct road between these points).

The first question is: weather to ask agent 1 to serve destination point B?

The second one is: when agent 2 passes this work, how to determine the charge (TC) for it?

4.2. Solution of the problem

First we're searching for the answer whether it is worth for agent 2 to pass the serving of destination point D to agent 1. Based on our method we have to determine the profit functions of the agents in case of destination point pass for finding the answer. The two possibilities of the transport are illustrated in Figure 4. Figure 4a shows the case when agent 2 serves his own destination point B, while Figure 4b shows the case of destination point passing.

Let's see agent 2's profit function: OE – TC, where

- OE is equal to the transfer cost from D to B through Székesfehérvár and back to his repository minus the cost of the path from D to Székesfehérvár. (Subtraction is because agent 2 has to come one way path from D to Székesfehérvár in each case.) It is just the transfer cost from Székesfehérvár to point B and back: f(64 km).
- TC is the charge he pays for agent 1's work if he passes this task to her.

Agent 2's profit is OE - TC = f(64 km) – TC. It means, that it is worth to pass destination point B if agent 2 pays less than f(64 km) for the transfer to agent 1:

$$TC < f(64 \text{ km}).$$

Now agent 2 has to count agent 1's profit function to determine whether he has a possibility - and if he has what the conditions are – to pass destination point B.

Agent 1's profit is $P + TC - E_1$, where P is the price of the gas bottles she carries to the customer and E_1 is the value of agent 1's extra transport expenses. TC is the previously introduced income for her work (what is paid by agent 2).

Knowing that agent 1 is on the road and will just arrive to destination point A, agent 2 can calculate agent 1's extra transport expenses (E_1) if she takes over the serving of B. It means that agent 1 should choose path A-B-C instead of the direct A-C path. The extra kilometers are: 74+50-53=71 km, so E_1=f(71 km).

Based on this result agent 1's profit can be written as

$$P + TC - E_1 = P + TC - f(71 \text{ km}).$$

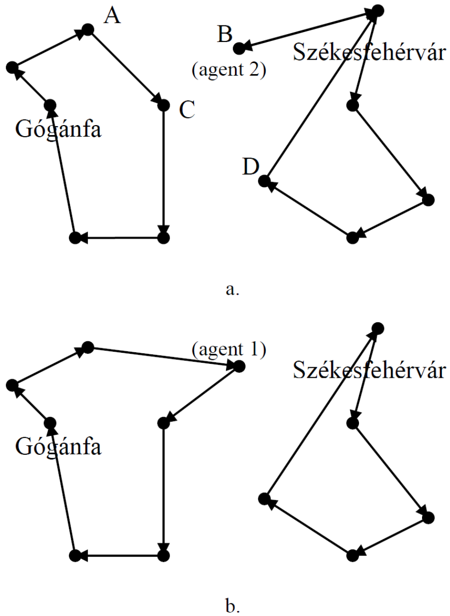

Figure 4: Case study: The two possible outcomes

Using the results we got from agent 2's profit ($\mathrm{TC} < \mathrm{f}(64\ \mathrm{km})$) we can determine an upper bound to agent 1's profit as follows:

$$\mathrm{P} + \mathrm{TC} - \mathrm{E1} = \mathrm{P} + \mathrm{TC} - \mathrm{f}(71\ \mathrm{km}) < \mathrm{P} + \mathrm{f}(64\ \mathrm{km}) - \mathrm{f}(71\ \mathrm{km}) = \mathrm{P} - \mathrm{f}(7\ \mathrm{km}).$$

It means that all the situations that are worth for agent 2 result smaller profit for agent 1 than P − f(7 km).

Naturally the destination point transfer is worth for agent 1 if her profit is bigger than zero. That's why we can write:

$$\mathrm{P} - \mathrm{f}(7\ \mathrm{km}) > 0, \text{ so } \mathrm{P} > \mathrm{f}(7\ \mathrm{km}).$$

It means that the agent from Gógánfa (agent 1) may serve destination point B if the price of the gas bottles is bigger than the cost of 7 km transfer.

At this point we've got the answer for the first question. The second question was how to determine value TC?

As we've determined, in case of non-cooperative players

$$\mathrm{TC} = \mathrm{OE} - \varepsilon = \mathrm{f}(64\ \mathrm{km}) - \varepsilon.$$

This situation is worth for both agents and there is no player which may have bigger outcome if he changes his strategy unilaterally. It means that it is the Nash-equilibrium of this game.

So the result for competitive agents is that agent 2 passes the serving of customer in destination point B to agent 1 and pays a bit smaller amount of money for her than the transport cost of 64 km.

The decision modifies a bit in case of cooperative suppliers. When the agents work for the same company, the goal is to minimize the overall expenses while supplying the 10 customers. In case of cooperative players the length of the extra path is 64 km in case of agent 2 serves destination point B and 71 km in case of agent 1 supplies customer in point B. It means that the overall cost is minimized when agent 2 serves destination point B.

4.3. Summary of the solution

We have seen that our method is working in a real-life situation. However for the sake of distinctness and simplicity we dealt only with a small portion of a whole fleet and customer set. In our solution there was no need for using the service of a central dispatcher center, in case of real-time changes the agents can make their own decisions themselves. It was also interesting to see that different solutions were born for cooperative and non-cooperative cases. In our future work we plan to extend our method to a whole region of the country and test it in realty.

5. Conclusion

In this paper we gave a method for transportation problems, where several destination points' needs have to be satisfied by suppliers. First we gave an algorithm for initial destination point separation. We discussed both cooperative and non-cooperative cases. Afterwards we analyzed the impact of a sudden change in the circumstances which may influence the destination point separation. We collected the parameters of the agents and based on game theory described the suppliers profit functions. Using the profit functions we answered the questions how to modify the initial customer separation if it is needed and how to determine the financial effects. In this part we also dealt both with cooperative and competitive cases. Finally we showed that our theory is working in a real-life problem.

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