Proceedings of the 8th International Conference on Applied Informatics
Eger, Hungary, January 27–30, 2010. Vol. 1. pp. 447–454.

Computer Aided Geometric Design of Gear Surfaces*

László Dudás

Department of Information Technology, University of Miskolc
e-mail: iitdl@uni-miskolc.hu

Abstract

The paper presents the Surface Constructor (SC) 3D kinematical modelling and simulation program, which is intended for the development of new types of complex contacting kinematical surfaces, especially gear surfaces. This innovative application is based on an original theory of the author, named the Reaching Model. The paper summarises the theory and introduces the structure and capabilities of the software. The versatility of SC, which origins from its inbuilt symbolic algebraic computation module, is presented through development and analysis of new gear types and by solution of manufacturing tasks of gearing elements, such as the calculation of a ZTA type worm gear. The power of SC is demonstrated through the determination of the grinding wheel for exact grinding of a spiroid worm and modelling of a double-modified ZTA type worm gearing, in addition to the simulation of an innovative new worm gearing type having localised bearing pattern without transmission error.

Keywords: gears, Reaching Model, Surface Constructor, intermediary generating surface

MSC: 51N04, 68N04, 68W04

1. Introduction

The quality of gears and efficiency of gear connections has been a major interest of gear designers and manufacturers from the start [1,6,7]. The special characteristics and requirements of this goal have resulted in special software tools for aiding the processes of design and technology. There are many unique software programs for design of a given type or narrow set of types of commonly applied gearings. These programs are created and used by gear manufacturing machine tool providers or

*Supported by the TAMOP-4.2.1-08/1-2008-0006 project. The project takes place through the support of the European Union, co-financed by the European Social Fund.

gearing design offices. There are fewer applications for helping with the innovation of new gearing types [8].

Figure 1 gives a comparison of some of the known gearing design applications [3]. This comparison reviews the capabilities of the programs for dimensioning or for optimization and development of new gearings. Furthermore, it shows the approximate number of embedded gearing kinematics as the freedom of applicability. Analysing the characteristics of these applications, it becomes conspicuous that the freedom of modelled gear types is limited. To eliminate this restriction the author developed the Surface Constructor application for modelling gear connections and aiding the design and analysis of mating surfaces.

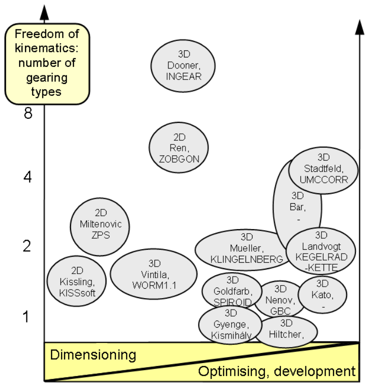

Figure 1: Comparison of gearing design programs

2. Theoretical background: The Reaching Model

The applied theory can be understood with the help of Figure 2a. The task is to determine the F2 surface, which is enveloped by the F1 in the relative motion. The $P'_k, ..., P'_r$ points of the F2 can be determined as the starting points of those paths of motion that are tangential to surface F1. To determine P'_k the search process starts with the path of motion of $R = 0$ and, moving from path to path in R direction, approaches F1. The path of motion that reaches F1 first will determine the P'_k point. Repeating the process by T results in the $P'_k, ..., P'_r$ points of the searched F2 surface in the $\kappa_l = \kappa_l(\Phi, R, T)$ curved co-ordinate system. Repeating the process by $Z, (Z = l, ..., s)$ using new κ co-ordinate systems, the F2 surface can be generated as a $T - Z$ grid. One of the main advantages of the method is its simplicity: determining an F2 point is equal to solving a minimum value problem, as can be seen in Figure 2b. The second advantage of the method that all the undercut situations can be identified discussing the minimum value problem, as can be seen in Figure 3. The third advantage is that the local-global extremum duality can be interpreted as (local) undercut and (global) cut. Figure 4 shows an example of global cut. The SC application exploits the possibilities of $R = R(\Phi)$ functions to give a tool to detect and avoid undercut regions of F2. This tool is a window that can show the $R = R(\Phi)$ function at every $T - Z$ point of F2, or can visualise a set of functions as a surface at a given T or given Z value. The precondition of undercut-free F2 surface is that the functions have an internal minimum point. Inflexion or horizontal function curves indicate the presence of undercut. A full discussion is given in [2].

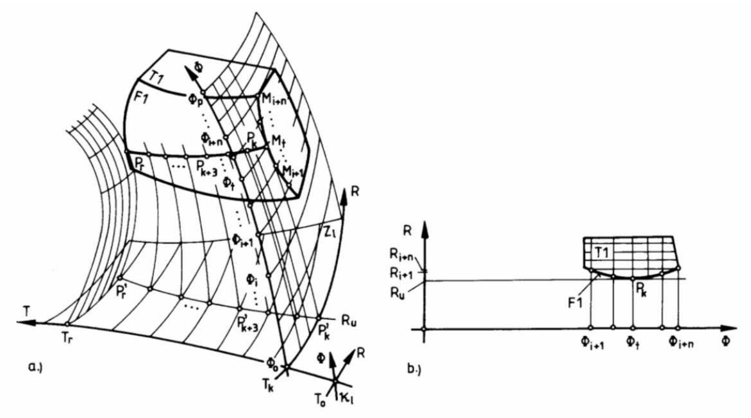

Figure 2: The reaching process

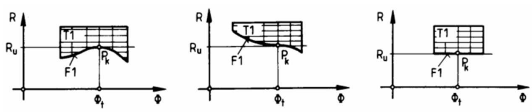

Figure 3: Situations of local undercut

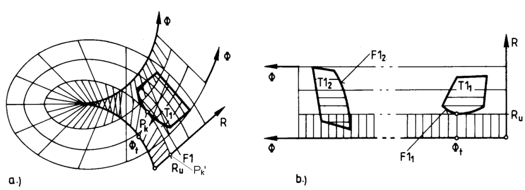

Figure 4: The global cut destroys the enveloped P_k' point

3. The Surface Constructor software application

The software actualizes the Reaching Model theory and makes it possible to model surface generation by surface, to animate kinematical relations and to optimise gear connection properties. The main goal, to provide maximal freedom in entering generating surfaces and constructing kinematical arrangements, was achieved by an inbuilt symbolic algebraic computation module [5]. The application works similarly to a CAD program, but it aids entering curves and surfaces by expressions and the relations among frames can be parameterized by expressions determining rotations and translations. The structure of SC mirrors these, as Figure 5 shows. The first level handles symbolic input and performs symbolic vector and matrix multiplications and inversions, the second represents the numeric forms of the surfaces and kinematics, while the third level visualises these objects. Detailed explanation of SC can be found in [3,4]. As a special capability, the F1 generating surface can act as a theoretical intermediary surface enveloping $F2_1$ and $F2_2$ surfaces. While F1-$F2_1$ or F1-$F2_2$ are conjugate pairs contacting in line, $F2_1$ and $F2_2$ contact in a point.

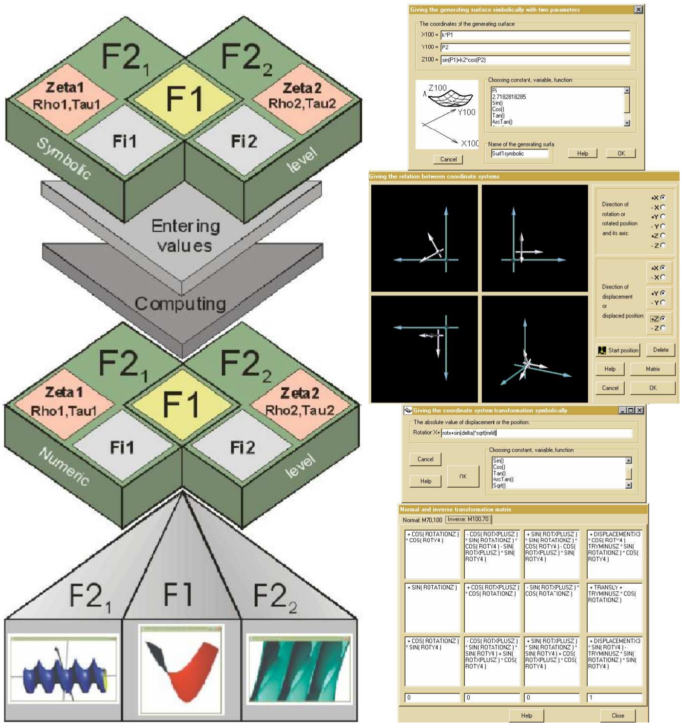

Figure 5: Left: The structure of SC Right: Steps of symbolic computation

4. Sample tasks

The following examples demonstrate the freedom provided by SC in modelling and analysis of gear and tool surfaces.

4.1. Modelling a ZTA type worm gearing

This example, shown in Figure 6, presents calculated conjugate worm and generated worm wheel gearing elements. The worm gear has a large undercut region. The machined worm gear proves the accuracy of the surface generation.

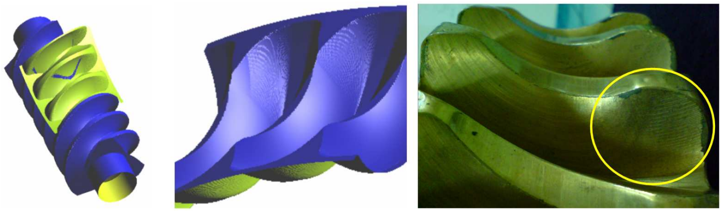

Figure 6: The ZTA worm gearing with the contact line, the calculated worm wheel segment and the manufactured real worm wheel surface

4.2. Exact grinding of a spiroid worm

The grinding of worms that have changing diameter along their axis is difficult because of the changing contact line between the worm and the grinding wheel. As the surface of a revolution shape grinding wheel has a given profile, it is not suitable for grinding such worms as globoid, having an hourglass form or spiroid, having a conical shape. The solution of theoretically exact grinding is suggested in an invention of the author. This method uses a special grinder machine and the grinding tool is also special: the conjugate surface of the worm, as shown in Figure 7.

4.3. Double-modified localised worm gearing

The theoretically exact size and relative position determined in the design process of a worm and worm wheel is difficult to realise because of manufacturing and assembly tolerances. A small difference can destroy the line contact [9]. To avoid this and make the gearing more error-tolerant, instead of line contact a smaller contact pattern localised to the inner region of the tooth is proposed. To achieve this, the author applied a special pitch modification along the worm axis and a second modification in radial direction of the teeth. As a result, the working of the gearing will be smooth and can tolerate small errors. The cubic function of the

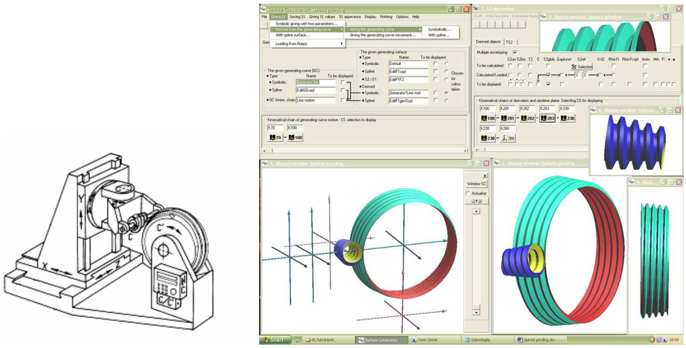

Figure 7: Grinding of spiroid worm needs special grinding machine and wheel

modified pitch is shown in Figure 8a, while the second modification using a larger ellipse arc to localise the connection in radial direction of the worm is presented in Figure 8b. The first modification guarantees the smooth entering and exiting of

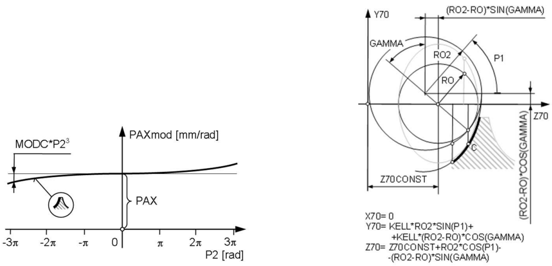

Figure 8: The two modifications applied on the same worm
a) The cubic function of the pitch modification b) Modification in radial direction

the worm teeth in the course of the meshing at the two ends of the worm, while the second modification provides smooth coupling of the teeth in the middle of the worm, as can be confirmed in Figure 9. Figure 10 shows the resulted localised contact patterns. These modifications result in low noise and vibration [8].

4.4. Worm gearing with point-like contact

This example solves the problem of modified worm gearings. The profile or tooth modification applied to achieve noiseless and misalignment-tolerant worm gearings usually results in error in transmission [10]. The use of an intermediary generating surface for the generation of worm gearing members solves this problem, because it

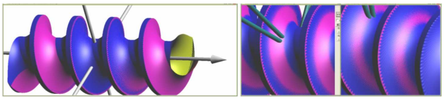

Figure 9: The worm with axially modified tooth surfaces and the lighter reference helicoid (to the left). Different radial modifications on the worm (to the right).

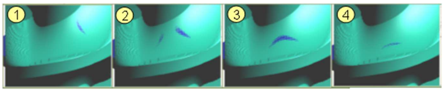

Figure 10: Contact patterns in different phases of connection

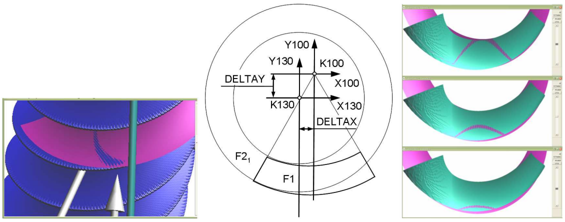

Figure 11: The contact line between $F2_1$ worm surface and F1 has to intersect every contact line between F1 and $F2_2$ worm gear surface

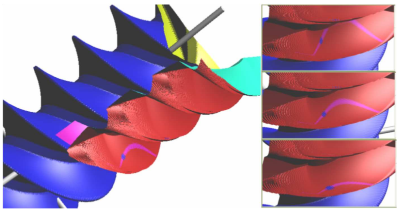

Figure 12: Point-like contact between the worm and the worm wheel

gives transmission-error-free continuous point-like contact between the worm and the worm wheel. Figure 11 shows the theoretical generating helicoid surface as F1 generating surface, the generated $F2_1$ worm surface with the contact line in a fixed position and the other generated surface, the $F2_2$ worm wheel tooth surface. The left side of Figure 12 shows the worm, the segment of the worm wheel, the intermediary generating helicoid segment and the contact arc between the worm wheel and the generating helicoid. The right hand side shows three moments of the contact patch wandering. The dark patch on the arc is the momentary point-like contact pattern.

5. Summary

This paper reviewed some results achieved using the Surface Constructor gears connection modelling application. The importance of the Reaching Model theory as the foundation of the tool was discussed. Then a short description presented the structure of the software. The emphasised versatility originates from the symbolic algebraic calculation, which is one of the special capabilities of the software. In the main part of the paper, some solved tasks were shown. These examples prove the applicability of SC for gear investigation and innovation.

References

[1] Dudás, I., Theory and practice of worm gear drives, *Penton Press London* (2000)

[2] Dudás, L., A consistent model for generating conjugate surfaces and determining all the types of local undercuts and global cut, *Proceedings of UMTIK'96 International Machine Design and Production Conference*, Ankara, (1996), 467–476.

[3] Dudás, L., Surface Constructor - a tool for investigation of gear surface connection, *Proceedings of CIM 2003 Advanced Design and Management Conference*, Wydawnictwa Naukowo-Technicne, Warszava, (2003), 140–147.

[4] Dudás, L., New theory and application for development of gear drives, *Proceedings of X. International Conference on the Theory of Machines and Mechanisms*, Liberec, (2008), 193–198.

[5] Goos G., Hartmanis J., Lecture notes in computer science 72. Symbolic and Algebraic Computation, *Springer-Verlag*, (1979)

[6] Litvin, F. L., Gear geometry and applied theory, *Prentice Hall, Englewood Cliffs, NY*, (1994)

[7] Litvin, F. L., Fuentes, A., Gear geometry and applied theory, *Cambridge University Press, Cambridge*, (2004)

[8] Lunin, S., Design of worm gears for low noise applications using worm gear software from ZAKGEAR, *http://www.zakgear.com/Worm.html*, Accessed: 20 Nov. 2009

[9] Ohsima, F., Yoshino, H., Ideal tooth surface modification method for cylindrical worm gears, *Proceedings of the JSME International Conference on Motion and Power Transmissions*, VT:MPT II(01-202), (2001), 460–465.

[10] Seol, I., Chung, S., Design and simulation of meshing of new type of worm-gear drive with localized contacts, *KSME International Journal*, Vol. 14, No. 4 (2000), 408–417.

László Dudás
Miskolc-Egyetemváros, H-3515 Miskolc, Hungary