Proceedings of the 7^{th} International Conference on Applied Informatics
Eger, Hungary, January 28–31, 2007. Vol. 2. pp. 7–14.

Teaching linear algebra with MatLab

Sándor Buglyó

Faculty of Engineering University of Debrecen
e-mail: buglyo@freemail.hu

Abstract

MatLab is used worldwide as a supportive tool in teaching mathematics (especially in higher technical education). Its merits include effective visual demonstration as well as the quick solution and revision of problems that would otherwise require a lot of calculation.

Providing the basics that are essential to understand the more advanced subjects of engineering we use MatLab. Considering certain time-consuming algorithms, from a methodical point of view, it seems reasonable to involve MatLab in the teaching of linear algebra for students of engineering and management [2].

In this paper, we provide a few examples to support and follow a possible approach to using MatLab teaching mathematics.

Keywords: MatLab

MSC: 97Cso

1. Introduction

There are to lot of changes in the education at the present time. The ratio of lessons in mathematics is decreasing because of a large number of technical subjects; in many cases the students' knowledge is not enough to follow the traditional educational process.

To answer these problems new ways have to be found in the education. The content has to be modified; suitable approaches have to be found provided by mathematical software [1]. In higher technical education MatLab is the most popular system for teaching/studying mathematics and different technical subjects. Furthermore knowing MatLab could come in handy even when looking for a job: MatLab is used by a reasonable number of companies today.

Our experience shows that thanks to the program's speed in completing basic operations, more problems can be solved in the same time; students can check their results instantly. Avoiding banal mistakes of calculation allows students to concentrate on solving the problem. The majority of students are more likely to favor this kind of approach. But it has several drawbacks e. g. up-to-date computer and software are required; students need to master a new language; syntactical errors can put back the calculation.

To show our developing activity in methodology of teaching mathematics (linear algebra) we present a few very simple examples. It seems to be clear that the more time-consuming an algorithm is, the more difficult to motivate the students to deal with that. For example calculating a determinant or an inverse matrix needs a large amount of fundamental operations. One can say that the result can be given by a simple command in a computer algebra system. However there is a problem: the learner can get the result by this way but can not understand the theoretical background and the details of the method. To enjoy the advantages of calculation by hand and usage of a computer we combine these methods: we apply the calculating ability of mathematical software to carry out only the steps of the algorithms. The tools of MatLab are specifically suitable for this goal.

2. Examples

Example 2.1. Calculating a determinant using basic transformations. Start with the matrix D and calculate its determinant.

```
D =
  2   3  -2   4
  3  -2   1   2
  3   2   3   4
 -2   4   0   5
D(1,:)=D(1,:)+2*D(2,:)  (first row = first row + 2* second row)
  8  -1   0   8
  3  -2   1   2
  3   2   3   4
 -2   4   0   5
D(3,:)=D(3,:)-3*D(2,:)   (third row = third row + 3* second row)
   8  -1   0   8
   3  -2   1   2
  -6   8   0  -2
  -2   4   0   5
D=D([1 3 4],[1 2 4]) (eliminate the second row and the third column)
```

The element in row 2 and column 3 of D is 1, and $2 + 3$ is odd, so

```
  suff(-1):
  8  -1   8
 -6   8  -2
 -2   4   5
D(:,1)=D(:,1)+8*D(:,2)  (first column = first column + 8* second column)
  0  -1   8
 58   8  -2
 30   4   5
```

`D(:,3)=D(:,3)+8*D(:,2)` *(third column = third column + 8∗ second column)*

```
  0  -1   0
 58   8  62
 30   4  37
```

The element in row 1 and column 2 of D is -1, and $1 + 2$ is odd, so (-1) times:

```
D =
 58  62
 30  37
```

The result is

```
-(58*37-30*62)
ans =
  -286
```

Check it with the command of `Det`:

```
Det(D)=
ans =
  -286
```

Example 2.2. Solving a linear system of equations by Cramer's rule. Let us see it is ".m" file.

```
format rat
clc
disp('Use the Cramer's rule to solve:')
disp('2x+y-2z=12')
disp('3x-2y+3z=7')
disp('2x+3y+3z=-7'),pause
A=[2 1 -2;3 -2 3;2 3 3]
B=[12;7;-7],pause
disp('[B,A(:,[2 3])] command will change the first column of A to the B
array')
[B,A(:,[2 3])]
disp('The next command: det([B,A(:,[2 3])])/det(A) and we get the result
for x')
x=det([B,A(:,[2 3])])/det(A),pause
disp('det([A(:,1),B,A(:,3)])/det(A) similarly for y ')
[A(:,1),B,A(:,3)]
y=det([A(:,1),B,A(:,3)])/det(A),pause
disp('det([A(:,[1 2]),B])/det(A)and for z')
[A(:,[1 2]),B]
z=det([A(:,[1 2]),B])/det(A)
```

Example 2.3. Determining the inverse of a matrix:

```
  1  -1   1  -3
  1  -2   3  -4
```

```
   3   4  -1   2
  -2   3   2   1
B=[B,eye(4)]
   1  -1   1  -3   1   0   0   0
   1  -2   3  -4   0   1   0   0
   3   4  -1   2   0   0   1   0
  -2   3   2   1   0   0   0   1
B(2,:)=B(2,:)-B(1,:)            (second row = second row - first row)
   1  -1   1  -3   1   0   0   0
   0  -1   2  -1  -1   1   0   0
   3   4  -1   2   0   0   1   0
  -2   3   2   1   0   0   0   1
B(3,:)=B(3,:)-3*B(1,:)          (third row = third row - 3*first row)
   1  -1   1  -3   1   0   0   0
   0  -1   2  -1  -1   1   0   0
   0   7  -4  11  -3   0   1   0
  -2   3   2   1   0   0   0   1
B(4,:)=B(4,:)+2*B(1,:)
   1  -1   1  -3   1   0   0   0
   0  -1   2  -1  -1   1   0   0
   0   7  -4  11  -3   0   1   0
   0   1   4  -5   2   0   0   1
B(2,:)=B(2,:)+B(4,:)
   1  -1   1  -3   1   0   0   0
   0   0   6  -6   1   1   0   1
   0   7  -4  11  -3   0   1   0
   0   1   4  -5   2   0   0   1
B(3,:)=B(3,:)-7*B(4,:)          (third row = third row - 7*fourth row)
   1  -1   1  -3   1   0   0   0
   0   0   6  -6   1   1   0   1
   0   0 -32  46 -17   0   1  -7
   0   1   4  -5   2   0   0   1
B(1,:)=B(1,:)+B(4,:)
   1   0   5  -8   3   0   0   1
   0   0   6  -6   1   1   0   1
   0   0 -32  46 -17   0   1  -7
   0   1   4  -5   2   0   0   1
B=B([1 4 2 3],:)                (change the order of rows)
   1   0   5  -8   3   0   0   1
   0   1   4  -5   2   0   0   1
   0   0   6  -6   1   1   0   1
   0   0 -32  46 -17   0   1  -7
B(3,:)=B(3,:)/6
   1   0   5  -8   3   0   0   1
   0   1   4  -5   2   0   0   1
   0   0   1  -1 1/6 1/6   0 1/6
   0   0 -32  46 -17   0   1  -7
```

```
B(1,:)=B(1,:)-5*B(3,:)
  1   0   0  -3 13/6 -5/6   0  1/6
  0   1   4  -5    2    0   0    1
  0   0   1  -1  1/6  1/6   0  1/6
  0   0 -32  46  -17    0   1   -7
```

```
B(2,:)=B(2,:)-4*B(3,:)
  1   0   0  -3 13/6 -5/6   0  1/6
  0   1   0  -1  4/3 -2/3   0   1/3
  0   0   1  -1  1/6  1/6   0   1/6
  0   0 -32  46  -17    0   1    -7
B(4,:)=B(4,:)+32*B(3,:)
  1   0   0  -3  13/6 -5/6   0 1/6
  0   1   0  -1   4/3 -2/3   0 1/3
  0   0   1  -1   1/6  1/6   0 1/6
  0   0   0  14 -35/3 16/3   1-5/3
B(4,:)=B(4,:)/14                      (fourth row/14)
  1   0   0  -3 13/6 -5/6    0   1/6
  0   1   0  -1  4/3 -2/3    0   1/3
  0   0   1  -1  1/6  1/6    0   1/6
  0   0   0   1 -5/6 8/21 1/14 -5/42
B(1,:)=B(1,:)+3*B(4,:)
  1   0   0   0 -1/3 13/42 3/14 -4/21
  0   1   0  -1  4/3  -2/3    0   1/3
  0   0   1  -1  1/6   1/6    0   1/6
  0   0   0   1 -5/6  8/21 1/14 -5/42
B(2,:)=B(2,:)+B(4,:)
  1   0   0   0 -1/3 13/42 3/14 -4/21
  0   1   0   0  1/2  -2/7 1/14  3/14
  0   0   1  -1  1/6   1/6    0   1/6
  0   0   0   1 -5/6  8/21 1/14 -5/42
B(3,:)=B(3,:)+B(4,:)
  1   0   0   0 -1/3 13/42 3/14 -4/21
  0   1   0   0  1/2  -2/7 1/14  3/14
  0   0   1   0 -2/3 23/42 1/14  1/21
  0   0   0   1 -5/6  8/21 1/14 -5/42
```

The result:

```
BINV=
 -1/3  13/42  3/14  -4/21
  1/2   -2/7  1/14   3/14
 -2/3  23/42  1/14   1/21
 -5/6   8/21  1/14  -5/42
```

We can check the result using only one command:

```
B*BINV=
  1  0  0  0
```

```
 *  1  0  0
 *  *  1  *
 *  *  0  1
```

Where `*` character means: “almost” zero. So it is due to rounding error.

Example 2.4. Determining reduced row echelon form.
The original matrix

```
  1   3   5   2   4
 -1   0   1   1  -7
  0   1  -1  -2  -7
  0   2   7   5   4
```

The generic element = A(1,1) and divine with it.

```
  1   3   5   2   4
 -1   0   1   1  -7
  0   1  -1  -2  -7
  0   2   7   5   4
```

Execute the transformation

```
  1   3   5   2   4
  0   3   6   3  -3
  0   1  -1  -2  -7
  0   2   7   5   4
```

The generic element = A(2,2)

```
  1   3   5   2   4
  0   1   2   1  -1
  0   1  -1  -2  -7
  0   2   7   5   4
```

Execute the transformation row-by-row (generic element = A(2,2))

```
  1   0  -1  -1   7
  0   1   2   1  -1
  0   1  -1  -2  -7
  0   2   7   5   4

  1   0  -1  -1   7
  0   1   2   1  -1
  0   0  -3  -3  -6
  0   2   7   5   4

  1   0  -1  -1   7
  0   1   2   1  -1
  0   0  -3  -3  -6
  0   0   3   3   6
```

The generic element = A(3,3) and divine with it.

```
  1    0   -1   -1    7
  0    1    2    1   -1
  0    0    1    1    2
  0    0    3    3    6
```

Execute the transformation row-by-row

```
  1    0    0    0    9
  0    1    2    1   -1
  0    0    1    1    2
  0    0    3    3    6

  1    0    0    0    9
  0    1    0   -1   -5
  0    0    1    1    2
  0    0    3    3    6

  1    0    0    0    9
  0    1    0   -1   -5
  0    0    1    1    2
  0    0    0    0    0
```

4. column (generic element = 0) and 5. column (generic element = 0) so the result: We can also check the result using another one command:

```
rref(A)=
  1    0    0    0    9
  0    1    0   -1   -5
  0    0    1    1    2
  0    0    0    0    0
```

3. Conclusion

Presenting a simple example from the area of linear algebra, our aim was to show a possible way to use MatLab in the teaching process. The usage of a computer algebra system for general purpose in basic mathematics courses, in our experience, may causes problems: the students can not see the essentiality behind the notions and methods. This fact might encourage one to exclude these tools from courses in mathematics. In our opinion this is not reasonable, we can find the midway: we build up the method following its steps using a computer as a calculator. During this activity the learner can concentrate on the reason of the steps instead of trivial operations.

References

[1] Kocsis, I., Application of Maple ODE Analyser in the investigation of differential equations in the higher engineering education, *Pollack Periodica* (under publication).

[2] Kocsis, I., Sauerbier, G., Tiba, Zs., Die Nutzung des Computersystems MAPLE in der Ingenieurausbildung der Universität Debrecen, *UICEE Global Journal for Engineering Education*, Vol. 10., Melbourne-Wismar (2006).