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A visibility algorithm for the projection $PS^d \to PS^2$

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Abstract

In the previous conference (ICAI 2004, Eger) we suggested a general method for describing geometry of k-spaces in a d-dimensional real projective metric space and for the projection of PS^d onto PS^2. This method allows an approach to Euclidean, hyperbolic, spherical and other geometries uniformly and it determines the visibility of the d-dimensional solids.

Applying the general method we concretize the visibility algorithm by a simple vector calculus. We emphasize that we look for objects from the viewpoint, as centre, along a half-line, and in this sense a line has two ideal points at infinity.

We illustrate this method by the projection of four-dimensional cube. First we apply the usual axonometric projection onto the xy plane from the ideal points of third and fourth axes, then the central projection from two proper points and their lines, as centre figure, and finally we render the picture by our visibility method. Thus we look at the 2-surface of the 4-cube in front of us.

Later on, we plan to move with the centre line and with the image 2-plane, while the 4-cube stands, and apply this method to the other regular 4-solids (see the homepage of István PROK `http://www.math.bme.hu/~prok`), etc.

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1. Introduction

In the real d-dimensional projective sphere $PS^d(\underline{\mathbf{V}}^{d+1}, \overline{\boldsymbol{V}}_{d+1}, \mathbb{R})$ a point $X = (\underline{\mathbf{x}})$ is defined by a ray $(\underline{\mathbf{x}}) = \{\underline{\mathbf{y}} = c\underline{\mathbf{x}} : c \in \mathbb{R}^+\}$ $\underline{0} \neq \underline{\mathbf{x}} \in \underline{\mathbf{V}}^{d+1}$, as a real $(d+1)$-dimensional vector space; then $(\underline{\mathbf{x}}) = (\underline{\mathbf{y}})$ or $\underline{\mathbf{x}} \sim \underline{\mathbf{y}}$. A signed hypersphere or $(d-1)$-sphere or $(d-1)$-plane or hyperplane $u = (\overline{\boldsymbol{u}})$ is defined by a form class $(\overline{\boldsymbol{u}}) = \{\overline{\boldsymbol{v}} = \overline{\boldsymbol{u}}c : c \in \mathbb{R}^+\}$ $\overline{0} \neq \overline{\boldsymbol{u}} \in \overline{\boldsymbol{V}}_{d+1}$, as a dual (form) space to $\underline{\mathbf{V}}^{d+1}$; then $(\overline{\boldsymbol{u}}) = (\overline{\boldsymbol{v}})$ or $\overline{\boldsymbol{u}} \sim \overline{\boldsymbol{v}}$. The point $(\underline{\mathbf{x}})$ lies on hyperplane $(\overline{\boldsymbol{u}})$, or in other words, the hyperplane $(\overline{\boldsymbol{u}})$ passes through the point $(\underline{\mathbf{x}})$, if $(\underline{\mathbf{x}}\overline{\boldsymbol{u}}) = 0$. The projective space P^d will be special case where opposite rays $(\underline{\mathbf{x}})$ and $(-\underline{\mathbf{x}})$ will be identified, so as $(\overline{\boldsymbol{u}})$ and $(-\overline{\boldsymbol{u}})$ are identified for hyperplanes. Affine space A^d and other non-Euclidean projective metric spaces can be modelled by PS^d in a unified way [1, 6].

2. Projection onto hyperplane $(\overline{\boldsymbol{u}})$ from the centre $(\underline{\mathbf{c}})$

2.1. Computing projection points

When $(\underline{\mathbf{y}})$ is the image of $(\underline{\mathbf{x}})$ from the centre $(\underline{\mathbf{c}})$ on the hyperplane (hypersphere) $\Pi = (\overline{\boldsymbol{u}})$, then $(\underline{\mathbf{y}})$ is a linear combination of the $(\underline{\mathbf{x}})$ and $(\underline{\mathbf{c}})$: (see Figure 1 and 3.)

$$\underline{\mathbf{y}} \sim (\underline{\mathbf{c}}\overline{\boldsymbol{u}})\underline{\mathbf{x}} - (\underline{\mathbf{x}}\overline{\boldsymbol{u}})\underline{\mathbf{c}}, \qquad \text{where } (\underline{\mathbf{x}}) \neq (\underline{\mathbf{c}}), (-\underline{\mathbf{c}}) \text{ ; and } (\underline{\mathbf{c}}\overline{\boldsymbol{u}}) > 0 \text{ can be assumed.}$$

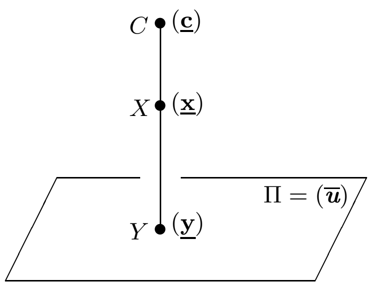

Figure 1: Projection onto hyperplane $\Pi = (\overline{\boldsymbol{u}})$ from the centre $C = (\underline{\mathbf{c}})$

If $(\underline{\mathbf{x}}_0), (\underline{\mathbf{x}}_1), \ldots, (\underline{\mathbf{x}}_k)$ span a k-plane denoted formally by $(\underline{\mathbf{x}}_0 \wedge \underline{\mathbf{x}}_1 \wedge \ldots \wedge \underline{\mathbf{x}}_k)$, then the images $(\underline{\mathbf{y}}_0), (\underline{\mathbf{y}}_1), \ldots, (\underline{\mathbf{y}}_k)$ determine

$$\underline{\mathbf{y}}_0 \wedge \underline{\mathbf{y}}_1 \wedge \ldots \wedge \underline{\mathbf{y}}_k \sim [(\underline{\mathbf{c}}\overline{\boldsymbol{u}})\underline{\mathbf{x}}_0 - (\underline{\mathbf{x}}_0\overline{\boldsymbol{u}})\underline{\mathbf{c}}] \wedge [(\underline{\mathbf{c}}\overline{\boldsymbol{u}})\underline{\mathbf{x}}_1 - (\underline{\mathbf{x}}_1\overline{\boldsymbol{u}})\underline{\mathbf{c}}] \wedge \ldots \sim$$

$$\sim (\underline{\mathbf{c}}\overline{\boldsymbol{u}})(\underline{\mathbf{x}}_0 \wedge \underline{\mathbf{x}}_1 \wedge \ldots \wedge \underline{\mathbf{x}}_k) - \sum_{i=0}^{k} (-1)^i (\underline{\mathbf{x}}_i \overline{\boldsymbol{u}})(\underline{\mathbf{c}} \wedge \underline{\mathbf{x}}_0 \wedge \underline{\mathbf{x}}_1 \wedge \ldots \wedge \underline{\mathbf{x}}_{i-1} \wedge \underline{\mathbf{x}}_{i+1} \wedge \ldots \wedge \underline{\mathbf{x}}_k),$$

if $(\underline{\mathbf{y}}_0 \wedge \underline{\mathbf{y}}_1 \wedge \ldots \wedge \underline{\mathbf{y}}_k)$ is not zero.

All these are only indicated here by the alternating (anti-symmetric) $\wedge$ (wedge) product in the Grassmann algebra of $\underline{\mathbf{V}}$ and of $\overline{\boldsymbol{V}}$, defined formally by induction on dimensions (see e.g. [5, 8]), as generalized Plücker coordinates. Determinants are important special cases. If it is zero, then we select independent vectors from $\underline{\mathbf{y}}_j'$s, say of index $0, 1, \ldots, r$ and form the $(r+1)$-vector $(\underline{\mathbf{y}}_0 \wedge \underline{\mathbf{y}}_1 \wedge \ldots \wedge \underline{\mathbf{y}}_r)$ above as an image r-plane of the k-plane. That case stands, if $\underline{\mathbf{c}} \in \underline{\mathbf{x}}_0 \wedge \underline{\mathbf{x}}_1 \wedge \ldots \wedge \underline{\mathbf{x}}_k$, then $r < k$ holds. Now $\underline{\mathbf{x}} \wedge \underline{\mathbf{x}}_0 \wedge \underline{\mathbf{x}}_1 \wedge \ldots \wedge \underline{\mathbf{x}}_k = 0$ means that the point $(\underline{\mathbf{x}})$ is incident to the k-plane $(\underline{\mathbf{x}}_0 \wedge \underline{\mathbf{x}}_1 \wedge \ldots \wedge \underline{\mathbf{x}}_k)$, in general.

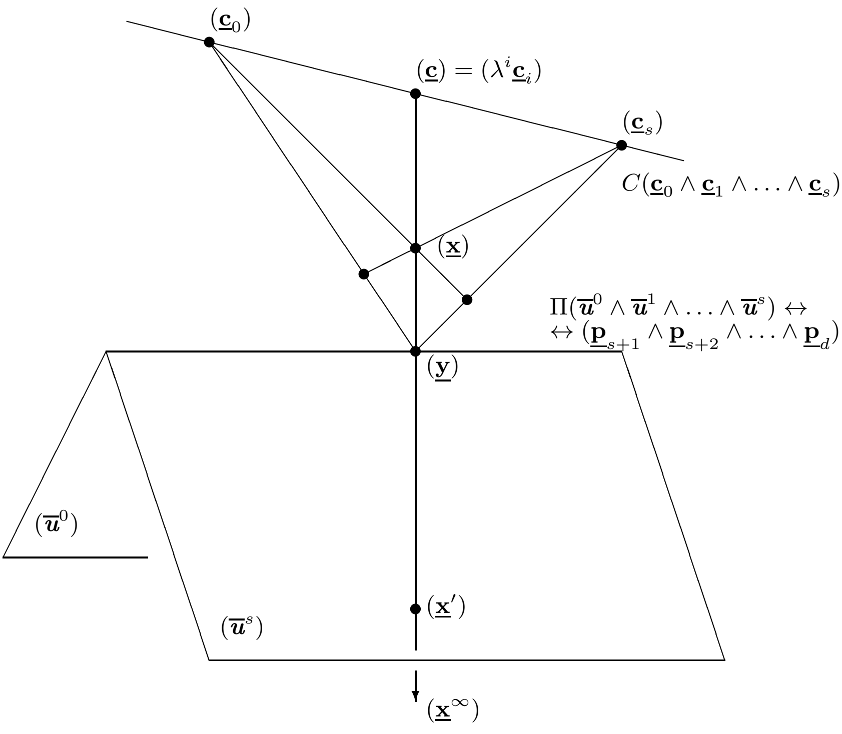

Figure 2: Finding the image and the centre of projection

Now let $C(\underline{\mathbf{c}}_0 \wedge \underline{\mathbf{c}}_1 \wedge \ldots \wedge \underline{\mathbf{c}}_s)$ be a centre s-plane and $\Pi(\overline{\boldsymbol{u}}^0 \wedge \overline{\boldsymbol{u}}^1 \wedge \ldots \wedge \overline{\boldsymbol{u}}^s)$ the image screen $(d-s-1)$-plane of a projection in our first interpretation. In the second interpretation the image $(d-s-1)$-plane Π will equivalently be given by $d-s$ points $(\underline{\mathbf{p}}_{s+1}), (\underline{\mathbf{p}}_{s+2}), \ldots, (\underline{\mathbf{p}}_d)$ and by their $(d-s)$-vector $(\underline{\mathbf{p}}_{s+1} \wedge \underline{\mathbf{p}}_{s+2} \wedge \ldots \wedge \underline{\mathbf{p}}_d)$, of course with $(\underline{\mathbf{p}}_i \overline{\boldsymbol{u}}^j) = 0$, $j \in \{0, 1, \ldots, s\}$, $i \in \{s+1, s+2, \ldots, d\}$. (Think of $d = 4, s = 1, d - s - 1 = 2$). The above $(s+1)$-form $(\overline{\boldsymbol{u}}^0 \wedge \overline{\boldsymbol{u}}^1 \wedge \ldots \wedge \overline{\boldsymbol{u}}^s)$ and $(d-s)$-vector $(\underline{\mathbf{p}}_{s+1} \wedge \underline{\mathbf{p}}_{s+2} \wedge \ldots \wedge \underline{\mathbf{p}}_d)$ are called dual to each other.

$(\underline{\mathbf{x}} \wedge \underline{\mathbf{c}}_0 \wedge \underline{\mathbf{c}}_1 \wedge \ldots \wedge \underline{\mathbf{c}}_s)(\neq 0)$ determines an $(s+1)$-plane whose intersection with the $(d-s-1)$-plane $(\overline{\boldsymbol{u}}^0 \wedge \overline{\boldsymbol{u}}^1 \wedge \ldots \wedge \overline{\boldsymbol{u}}^s)$ will be the image $(\underline{\mathbf{y}})$ of the point $(\underline{\mathbf{x}})$. See Figure 2 and 3 for $d = 3$, $s = 1$.

From $\underline{\mathbf{y}} \sim -\lambda^0 \underline{\mathbf{c}}_0 - \lambda^1 \underline{\mathbf{c}}_1 - \ldots - \lambda^s \underline{\mathbf{c}}_s + \underline{\mathbf{x}}$ such that

$$\begin{array}{ccccc}
0 & = & (\underline{\mathbf{y}}\overline{\boldsymbol{u}}^0) & = & -\lambda^0(\underline{\mathbf{c}}_0\overline{\boldsymbol{u}}^0) - \lambda^1(\underline{\mathbf{c}}_1\overline{\boldsymbol{u}}^0) - \ldots - \lambda^s(\underline{\mathbf{c}}_s\overline{\boldsymbol{u}}^0) + (\underline{\mathbf{x}}\overline{\boldsymbol{u}}^0) \\
\vdots & & \vdots & & \\
0 & = & (\underline{\mathbf{y}}\overline{\boldsymbol{u}}^s) & = & -\lambda^0(\underline{\mathbf{c}}_0\overline{\boldsymbol{u}}^s) - \lambda^1(\underline{\mathbf{c}}_1\overline{\boldsymbol{u}}^s) - \ldots - \lambda^s(\underline{\mathbf{c}}_s\overline{\boldsymbol{u}}^s) + (\underline{\mathbf{x}}\overline{\boldsymbol{u}}^s)
\end{array}$$

we get an inhomogeneous linear equation system of order $s+1$ for λ^i $i = 0, 1, \ldots, s$. Thus the corresponding centre point $(\underline{\mathbf{c}}) = (\lambda^i \underline{\mathbf{c}}_i)$ in the centre s-figure can be determined, then the image $(\underline{\mathbf{y}})$ of $(\underline{\mathbf{x}})$ as well, by $(s+1) \times (s+1)$ determinants, as by Cramer rule it follows

$$\lambda^i = \frac{\det[(\underline{\mathbf{c}}_0\overline{\boldsymbol{u}}^j), \ldots, (\underline{\mathbf{c}}_{i-1}\overline{\boldsymbol{u}}^j), (\underline{\mathbf{x}}\overline{\boldsymbol{u}}^j), (\underline{\mathbf{c}}_{i+1}\overline{\boldsymbol{u}}^j), \ldots, (\underline{\mathbf{c}}_s\overline{\boldsymbol{u}}^j)]}{\det[(\underline{\mathbf{c}}_0\overline{\boldsymbol{u}}^j), \ldots, (\underline{\mathbf{c}}_{i-1}\overline{\boldsymbol{u}}^j), (\underline{\mathbf{c}}_i\overline{\boldsymbol{u}}^j), (\underline{\mathbf{c}}_{i+1}\overline{\boldsymbol{u}}^j), \ldots, (\underline{\mathbf{c}}_s\overline{\boldsymbol{u}}^j)]},$$

where $\det(\underline{\mathbf{c}}_i\overline{\boldsymbol{u}}^j) > 0$ can be assumed.

Thus

$$\underline{\mathbf{y}} \sim \sum_{i=0}^{s} -\det[(\underline{\mathbf{c}}_0\overline{\boldsymbol{u}}^j), \ldots, (\underline{\mathbf{c}}_{i-1}\overline{\boldsymbol{u}}^j), (\underline{\mathbf{x}}\overline{\boldsymbol{u}}^j), (\underline{\mathbf{c}}_{i+1}\overline{\boldsymbol{u}}^j), \ldots, (\underline{\mathbf{c}}_s\overline{\boldsymbol{u}}^j)]\underline{\mathbf{c}}_i + \det[(\underline{\mathbf{c}}_0\overline{\boldsymbol{u}}^j), \ldots, (\underline{\mathbf{c}}_s\overline{\boldsymbol{u}}^j)]\underline{\mathbf{x}}$$

can be expressed! (Of course, by computer.)

2.2. Local visibility criterion

Assume that $(\underline{\mathbf{x}})$ and $(\underline{\mathbf{x}}')$ have the same image $(\underline{\mathbf{y}}) \sim ({}^p\underline{\mathbf{x}})$ with the same $(\underline{\mathbf{c}})$ in the centre s-figure. See Figure 3. The (distinguished) ideal point $(\underline{\mathbf{x}}^\infty)$ of the ray $(\underline{\mathbf{c}})(\underline{\mathbf{y}})$ will be described in the form $-\gamma\underline{\mathbf{c}} + \underline{\mathbf{y}} \sim \underline{\mathbf{x}}^\infty$, i.e. $(\underline{\mathbf{x}}^\infty \overline{\boldsymbol{e}}^0) = -\gamma(\underline{\mathbf{c}}\overline{\boldsymbol{e}}^0) + (\underline{\mathbf{y}}\overline{\boldsymbol{e}}^0) = 0$ by the linear form $(\overline{\boldsymbol{e}}^0)$ representing the ideal hyperplane at infinity of the Euclidean space $\mathbb{E}^d$. Thus $\underline{\mathbf{x}}^\infty \sim -(\underline{\mathbf{y}}\overline{\boldsymbol{e}}^0)\underline{\mathbf{c}} + (\underline{\mathbf{c}}\overline{\boldsymbol{e}}^0)\underline{\mathbf{y}}$. Then follows the local visibility criterion:

The $\underline{\mathbf{x}}' \sim \xi'\underline{\mathbf{c}} + \underline{\mathbf{x}}^\infty$ *is behind (covered by)* $\underline{\mathbf{x}} \sim \xi\underline{\mathbf{c}} + \underline{\mathbf{x}}^\infty$ *iff* $\xi > \xi' > 0$.

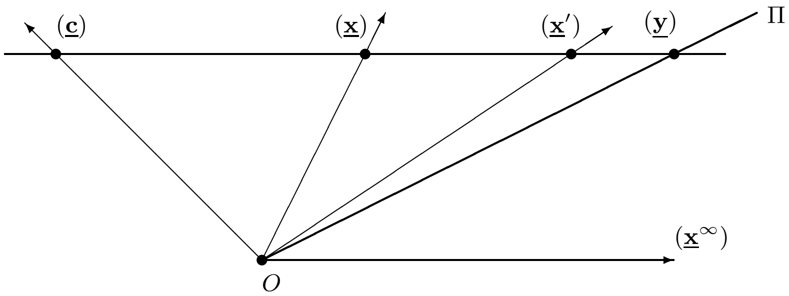

Figure 3: Two points with the same image by vector interpretation

2.3. Examples

To illustrate and test this method, we wrote a computer program, which projects a four-dimensional cube from an optional 1-dimensional centre (a line given by two points $(\underline{\mathbf{c}}_0)$ and $(\underline{\mathbf{c}}_1)$) to an optional two-dimensional plane given by two 3-planes by two forms $\overline{\boldsymbol{u}}^0, \overline{\boldsymbol{u}}^1 \in \overline{\boldsymbol{V}}_5$. Any point $(\underline{\mathbf{x}})$, not lying on the centre line, has a unique centre point $\underline{\mathbf{c}}$ and unique image point $(\underline{\mathbf{y}}) = ({}^p\underline{\mathbf{x}})$ so that $\underline{\mathbf{y}} \sim \underline{\mathbf{x}} + \underline{\mathbf{c}}$. Opposite to the most other applications in the literature, our program projects solids directly onto the two-dimensional screen, it does not use the three-dimensional space for intermediate step.

The cube is declared by an origin $(\underline{\mathbf{e}}_0)$ and by four base vectors $(\underline{\mathbf{e}}_1), (\underline{\mathbf{e}}_2), (\underline{\mathbf{e}}_3)$, $(\underline{\mathbf{e}}_4)$ (also as picking up the ideal points of axes) of the Cartesian homogeneous coordinate system. In Figure 4 we chose points $\underline{\mathbf{c}}_1 \sim (1,5,2,2,0)$ and $\underline{\mathbf{c}}_2 \sim (1,2,0,0,5)$ for centre of projection, and intersection of 3-planes $\overline{\boldsymbol{u}}^1 \sim \begin{pmatrix} 1 \\ 2 \\ 1 \\ 2 \\ 1 \end{pmatrix} =: (1,2,1,2,1)^T$ in transposed form and $\overline{\boldsymbol{u}}^2 \sim (1,2,1,1,2)^T$ as column matrices for picture 2-plane. Figure 5 was made by using the previous cube but different centres and 3-planes: $\underline{\mathbf{c}}_1 \sim (1, \frac{3}{4}, \frac{1}{4}, 4, 1)$ and $\underline{\mathbf{c}}_2 \sim (1, \frac{1}{2}, 2, \frac{1}{2}, 4)$ and $\overline{\boldsymbol{u}}^1 \sim (0,-2,2,-2,2)^T$ and $\overline{\boldsymbol{u}}^2 \sim (0,-3,-2,-3,-12)^T$. In this figure we applied our visibility algorithm.

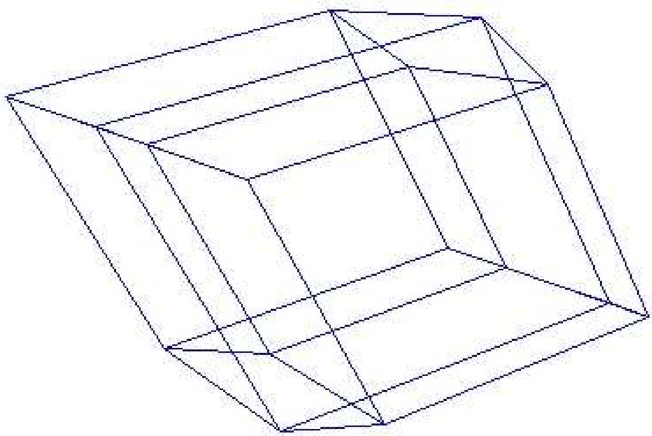

Figure 4: The frame of a four-dimensional cube

3. On global visibility for convex polytopes in $\mathbb{E}^4$

In this sketch we assume now a $d - s - 1 = p = 2$-dimensional picture plane Π, in Euclidean 4-space $\mathbb{E}^4$, given by a proper point $(\underline{\mathbf{p}}_0)$ and two ideal points $(\underline{\mathbf{p}}_1^\infty)$ and $(\underline{\mathbf{p}}_2^\infty)$, moreover an $s - 1 = 1$-dimensional centre figure C, given by a proper point $(\underline{\mathbf{c}}_3)$ and an ideal point $(\underline{\mathbf{c}}_4^\infty)$, all embedded in the projective 4-sphere $PS^4(\underline{\mathbf{V}}^5, \overline{\boldsymbol{V}}_5, \mathbb{R})$ as above.

A convex polytope P^4 (e.g. a 4-cube) is given by its edge framework. If $(\underline{\mathbf{y}}) = ({}^p\underline{\mathbf{x}}) = ({}^p\underline{\mathbf{x}}')$ is the intersection of the images of $(\underline{\mathbf{x}}_1)(\underline{\mathbf{x}}_2)$ and $(\underline{\mathbf{x}}_3)(\underline{\mathbf{x}}_4)$, then $(\underline{\mathbf{c}}) \in C$ and e.g. $(\underline{\mathbf{x}}) \in (\underline{\mathbf{x}}_1)(\underline{\mathbf{x}}_2)$ can be determined by $\underline{\mathbf{x}} = \xi^1\underline{\mathbf{x}}_1 + \xi^2\underline{\mathbf{x}}_2$ and the vector equation

$$\pi^0\underline{\mathbf{p}}_0 + \pi^1\underline{\mathbf{p}}_1^\infty + \pi^2\underline{\mathbf{p}}_2^\infty = \underline{\mathbf{y}} = (\xi^1\underline{\mathbf{x}}_1 + \xi^2\underline{\mathbf{x}}_2) + (\gamma^1\underline{\mathbf{c}}_3 + \gamma^4\underline{\mathbf{c}}_4^\infty) = \underline{\mathbf{x}} + \underline{\mathbf{c}}$$

as the inverse procedure to the former local visibility criterion (e.g. by Gauss elimination).

Then, to the contour of the polytope P^4, as to the convex hull of its projection, we determine the edges of the visible 2-faces of P^4 - to be described later on, in a forthcoming paper - which depend on the mutual positions of P^4 and of the centre figure C - or of the vanishing hyperplane, in general. Then we select the multiple image points and their edges, running to the contour or to the visible edges. We dot or cancel these covered edges and multiple points. Thus we obtain the 2-dimensional surface of P^4 in front of us (see e.g. Figure 5).

The ideal hyperplane $(\overline{\boldsymbol{i}})$ at infinity and the vanishing hyperplane $(\overline{\boldsymbol{v}})$ play important roles yet in this central projection. Namely $(\overline{\boldsymbol{i}}) = (\overline{\boldsymbol{e}}^0) = (\overline{\boldsymbol{e}}^{0'})$ will be

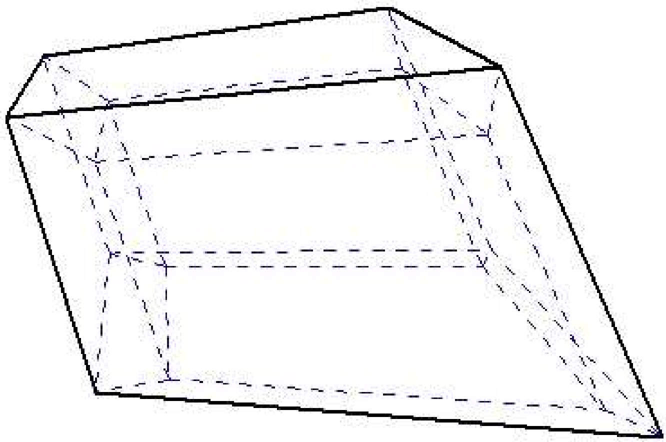

Figure 5: Visibility of a four-dimensional cube

chosen for every affine (or Cartesian) coordinate simplex, i.e. $x^0 = x^{0'} = 0$ holds for each ideal point $(\underline{\mathbf{x}}^\infty)$ at infinity. The vanishing hyperplane $(\overline{\boldsymbol{v}})$ (of disappearance) lies on the centre line $C = (\underline{\mathbf{c}}_3 \wedge \underline{\mathbf{c}}_4^\infty)$ parallel to the picture plane $\Pi = (\underline{\mathbf{p}}_0 \wedge \underline{\mathbf{p}}_1^\infty \wedge \underline{\mathbf{p}}_2^\infty)$. Hence $(\overline{\boldsymbol{v}})$ is expressed by the dual form to the 4-vector $(\underline{\mathbf{c}}_3 \wedge \underline{\mathbf{c}}_4^\infty \wedge \underline{\mathbf{p}}_1^\infty \wedge \underline{\mathbf{p}}_2^\infty)$.

Of course the sizes of picture screen also determine the visible part of $\mathbb{E}^4$ and so of P^4 between the vanishing 3-plane and the ideal 3-plane at infinity. But this problem is not detailed more in this conference report.

4. Further possibilities

We are working on an animation with moving projection centre figure and picture screen (camera position) and projects other optional objects (e.g. regular d-dimensional solids). Our reference list does not pretend completeness, of course.

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