

# On proving existence of some circle packings in a square using computer algebra systems

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## Abstract

The problem is the following: Locate  $n$  equal and non-overlapping circles in a square, such that the radius of the circles be maximal.

Up to  $n = 5$  circles the problem is trivial and there are solutions for  $n=6,8,9,14,16,25$  and 36 using only mathematical tools. Since 1990 real improvements were made in this field based on computer aided methods. Using deterministic optimization techniques, the optimal packings are known up to  $n = 30$  [3]. For higher  $n$  values (with the exception of  $n = 36$ , when the solution is known) stochastic methods (e.g. Threshold Accepting, billiard simulation [2], etc.) can be used to find good approximate packings.

It is important to realize that an approximate packing found by the computer is not always sure in the mathematical sense. The structure suggested by the numerical result is only a kind of conjecture, because the rounding errors can produce serious mistakes. We have to prove that the structure of a given packing really exists, that it is feasible.

A possible approach is based on algebraic, symbolic computations [1]. Find the corresponding suitable quadratic system of equations to the packing and try to solve it. In this case the computer algebra systems can help the investigation. The talk will show recent results in this field.

## References

- [1] P.G. Szabó, Optimal substructures in optimal and approximate circle packings, *Beitrage zur Algebra und Geometrie* (Accepted for publication).
- [2] P.G. Szabó and E. Specht, Packing up to 200 equal circles in a square, (Submitted for publication).
- [3] P.G. Szabó, M.Cs. Markót, and T. Csendes, Global optimization in geometry — Circle packing into the square (Submitted for publication).