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Projection by Grassmann algebra of E^4

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Abstract

In the analogy of Plücker's line coordinates a general method will be suggested for describing geometry of k-spaces in a d-dimensional real projective metric space and for the projection of E^d (as special case of P^d) onto a 2-dimensional screen E^2 .

Although the idea is classical, no traces appeared in the references about the topic (as far as I know). Indeed, the computer will be necessary to these computations, based on the Grassmann algebra of the projective spherical space $PS(V^{d+1}, V^{d+1}, R)$, endowed by the usual antisymmetric and multilinear wedge (exterior) product. This machinery gives a general framework for treating projective structure of k-spaces in P^d . E.g. the projection from an *s*-plane onto a d - s - 1-plane can be defined, then various collineations with *s*-centre and d - s - 1-axis can be discussed, etc.

We introduce a projective metric in P^d by giving a metric polarity as a symmetric linear map (*), or equivalently by a scalar product. Thus we associate a point $A = (a^*) = (a)$ as the pole for the polar hyperplane a = (a). Thus we can approach to Euclidean, hyperbolic, spherical and other geometries uniformly, just by the same machinery. Linear transforms, leaving the metric polarity invariant, define the groups of these geometries, respectively. Then metric invariants can be calculated as distance, angle, volume, etc. Hopefully, these and other applications will be illustrated in the lecture.