

Work-efficient prefix computation

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Abstract

Let $\Sigma = \{a_1, a_2, \dots, a_s\}$ be and let \oplus be a binary operation defined on Σ . We assume that \oplus is associative, that is $(x \oplus y) \oplus z = x \oplus (y \oplus z)$ for all $x, y, z \in \Sigma$. Such operations are addition, multiplication, minimum of two elements, concatenation of strings and so on.

Given elements $x_1, x_2, \dots, x_n \in \Sigma$, let S_{ij} denote the sum $x_i \oplus x_{i+1} \oplus \dots \oplus x_j$, $i \leq j$. The sums $S_{1j} = x_1 \oplus x_2 \oplus \dots \oplus x_j$, $j = 1, \dots, n$ are called *prefix sums* [1,2,3].

A parallel algorithm is called *work-efficient*, if its work has the same order as the work of the best known sequential algorithm.

There are parallel algorithms specified on CREW and EREW PRAM (parallel random access machine) computational models requiring n processors and $\Theta(\log n)$ time. Since the naive RAM algorithm runs $\Theta(n)$ time, the work ($\Theta(n \log n)$) of the mentioned algorithms has larger order and so these algorithms are not work-efficient.

We describe different CREW PRAM algorithms running on $f(n)$ PRAM processors and requiring $g(n)$ time so that $f(n)g(n) = \Theta(n)$ and analyse the bounds of the functions $f(n)$ and $g(n)$.

The manuscript of the paper can be found at

<http://people.inf.elte.hu/tony/publications/Parprefix.pdf>.

References

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