6th International Conference on Applied Informatics
Eger, Hungary, January 27–31, 2004.

Conformal application with computer graphics in simply connected regions*

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Abstract

The scope of the paper is the simulation of conformal application with computer graphics in simply connected regions. The problem is that: D and Δ domains are given. Determine the conformal application that transform the D domain in Δ.

I will present two different kinds of analysis of conformal application problem: the boundary point association by using the Lagrange interpolation formula, here I take different kind of distributions in the given domains boundary and the point pair association by going all over the given domains.

Categories and Subject Descriptors: 30C30 [Numerical methods in conformal mappings theory]

Key Words and Phrases: Conformal application, Lagrange interpolation, distributions, boundary points.

1. Introduction

In order to present my studies, we will need the following definitions and theorems presented in this section. In based on domain keeping principle we know that one holomorphic function the domain transforms into domain. By the examination of the geometrical meaning of complex derived, we get that if f is C^1 classes function, the real and the imaginary part of the function is continuous derivable, and $f'(z) \neq 0$.

Definition 1.1. Let G be open complex set and f a C^1 class function in G set. The f function in z_0 point is conformal of the first kind, if for every γ_1 and γ_2 smooth paths that start out from z_0, the angle of the $f \circ \gamma_1$ and $f \circ \gamma_2$ paths in $f(z_0)$ points is the same as the angle of γ_1 and γ_2 paths in z_0 point.

*Supported by the Research Programs Institute of Foundation Sapientia, Grant Nr. SZ/1765/08.12.2003.

Definition 1.2. The f in $z_0 \subset G$ point is conformal of second kind if every elementary arc length by way of f is changing in the same proportion, that is if

$$|(f \circ \gamma)'(0)| = k|\gamma'(0)|, \ \forall \text{ smooth}, \ \gamma(0) = z_0$$

and k number does not depend at γ. The k constant is the coefficient of linear deformation in z_0 point.

Thus the f function is conformal in every point at domain where the derived function is not zero. If f is injection in D domain, then f is a homeomorphism between D and $f(D)$, and f^{-1} is an injection in $f(D)$ domain. But we know that in D domain the holomorphic injection derived function is not zero in domain. Consequently the f function in D, while the f^{-1} function in $f(D)$ is conformal.

Definition 1.3. The f injection defined on D complex domain is conformal application if it transforms the D domain into $f(D)$.

Definition 1.4. The D and $f(D)$ domains are conformal equivalents, if there exists one conformal application that transforms the D domain into $f(D)$.

After this we can formulate the direct and indirect problems of conformal application.

1. The direct problem of conformal applications: D domain is given and f is a holomorphic injection in domain; we search the D domain image.

2. The indirect problem of conformal applications, what in fact is the conformal application problem: *D and Δ domains are given. Determine the conformal application that transforms the D domain in Δ.*

This kind of conformal application doesn't always exist, for example: if one of the domains is a simply connected region and the other isn't.

Further on I will present two theorems that I will use in the next section

Theorem 1.1. (Riemann theorem) *Every D complex simply connected domain is conformal equivalent with U, the unit radius disk.*

Theorem 1.2. *If we find a conformal application, than the set of D domain conformal application is given by*

$$f(z) = e^{i\theta} \frac{f_0(z) - a}{1 - \bar{a} f_0(z)} \ \textit{where } \theta \in \square, \ a \in U$$

type of function.

2. Methods for analyse the indirect problem of conformal applications

In this section I will study the direct problem of conformal application, the indirect problem connected to the Lagrange interpolation formula. Than I will show some distributions of the boundary points, which are uniform, cebisev and other distributions. And at the end I will present two discrete algorithms.

The first step of my research points, the direct problems of conformal applications, I've studied the unit radius disk or $(-1,1) \times (-1,1)$ domain image at some injections. Based on Riemann theory we generally choose the unit radius disk for study domain.

I formulated two questions regarding the indirect problems of conformal application.

1. How is the image at boundary points of the unit radius disk distributed in the boundary points of the given domain?

2. How does the first point pair selection influence the determination of association?

We know that in case of conformal application the boundary points go to boundary points and the interior of the domain goes to the interior points of the domain. For this I have studied only the boundary points of the domain.

In both cases we obtained the suspected answer, the unit radius circle points do not distribute uniformly in the boundary points of the given domain. In a first step I can assign any point on the boundary of unit radius disk to any point on the given domain boundary. For this question we used the Theorem 1.2. The figure 1 shore up these affirmations.

Figure 1: The $(z+2)^4$ conformal application

After this I turned to the indirect problems of conformal applications, i.e. two domains are given, and I want to receive the conformal application that, transforms one of the domain into the other. By using a property that f function is conformal in every point at domain where the derived function isn't zero we obtained that it's enough to look for one injection that transforms the unit radius disk image into the given domain. And the conformal application between two optional domains we find with the composite function of the conformal application which transforms the unit radius disk into the optional given domains.

The conformal application is a holomorphic function, it follows that infinite time is derivable, so we can give its Taylor's series, what is one polynomial. If we can give the Taylor's series of the conformal application in the pole, then we can determine the searched application. My first try was, by applying the Lagrange interpolation formula, to engross that complex polynomial that we can obtain from point pair association. We expect that as we extend the number of the point pair, the obtained polynomial will approach the Taylor's series of the wanted conformal application.

At first I use the information, that I already know, that is the searched conformal application, i.e. the point pair I choose in $(x, f(x))$ form, where x is a point on a unit radius circle, distributed uniformly.

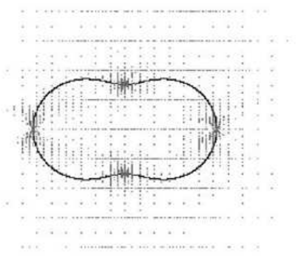

Figure 2: 4 point pair selection

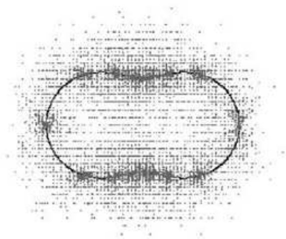

Figure 3: 16 point pair selection

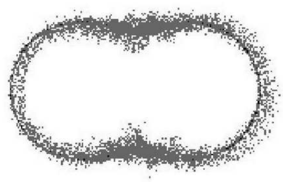

Figure 4: 600 point pair selection

These figures demonstrate the previous presented idea. Here the tested function was the tangent function, the way I increase the number of the point pairs the obtained polynomial function will approach the original figure.

But if I choose different points in a unit radius circle: $x_j = \cos(\varphi_j) + i\sin(\varphi_j)$, $j \in [-\pi, \pi]$ where $\varphi_j - \varphi_{j-1} = 0.1$ radian, we will obtain that with Lagrange interpolation formula we can characterise the boundary points of the domains.

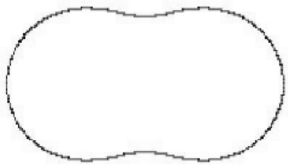

Figure 5: The tangent function

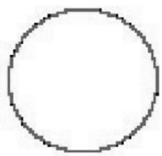

Figure 6: The $\log(z + 2)$ function

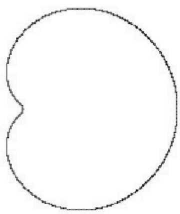

Figure 7: The $(z + 2)^4$ function

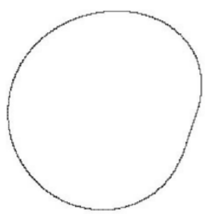

Figure 8: The $\frac{z+\sin(z)}{z+2i+3}$ function

If I don't know the searched conformal application, I have to choose even the distribution of the given domain boundary points. So the point pair are in (x, y) form. In this case the unit radius circle points, and the given domain boundary points are uniformly distributed.

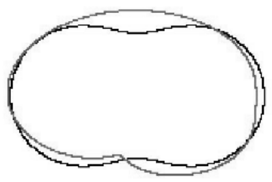
Figure 9: 4 point pair selection

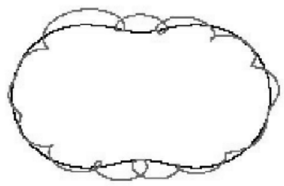
Figure 10: 16 point pair selection

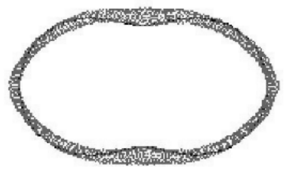
Figure 11: 200 point pair selection

Because the distribution of the points in a boundary is not uniform, the point pair association represents a difficulty. We can find the point pair of the original conformal application only with certain distribution, after we select the first point pair.

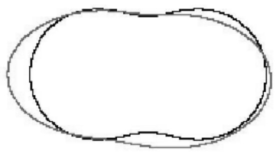
Figure 12: 4 point pair selection

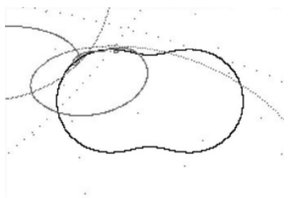
Figure 13: 16 point pair selection

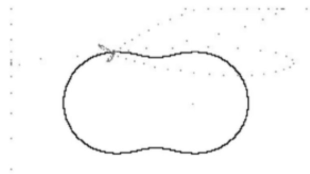
Figure 14: 200 point pair selection

Here the distribution is Cebisev in the unit radius circle, and in the given domain boundary. We know about Cebisev distribution that in case of real functions it is the best distribution. But in case of complex function it isn't.

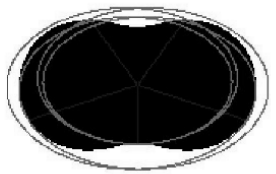
Figure 15: 4 point pair selection

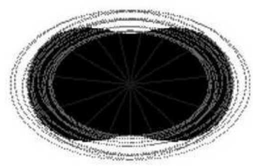
Figure 16: 16 point pair selection

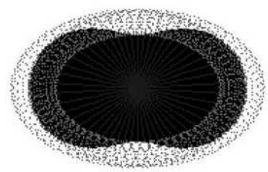
Figure 17: 60 point pair selection

In this case the distribution of unit radius circle is uniform. Based on these points the boundary points of the given domain (the intersection of the line that goes through the center of gravity and the chosen point of unit radius circle, with the given domain periphery) are chosen.

I tried even the case where the distribution in the unit radius circle is the same as I presented where the Lagrange formula works. The distributions of the given domain boundary points is one of the previously presented distributions.

Figure 18: Uniform distribution

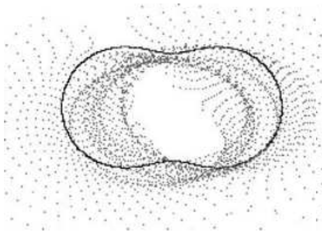

Figure 19: Cebisev distribution

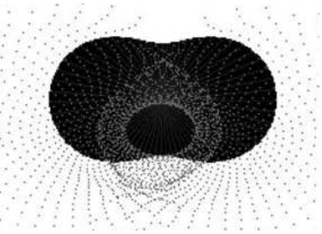

Figure 20: Other distribution

Further on I will present two discrete algorithms. The first type where the point color association algorithms is:

— we look for the center of gravity in the domains

— we go around the boundary points of the given domains

— we take the line that goes through the center of gravity and one point in the unit radius circle with origin in the center of gravity which intersects the boundary of the given domains

— we note the points that we find on k-th part of the segment line, what we have previously obtained

— by going around in the boundary points at the domains we obtain the k-th times of the original domain

— we associate the boundary points of these reduced-sized domains

Figure 21: First discrete algorithm result

And the second type of algorithms is:

— we look for the center of gravity in the domains

— we go around the boundary points of the given domains

— we take the line that goes through the center of gravity and one point in the unit radius circle with origin in the center of gravity which intersects the boundary of the given domains

— we associate the previously obtained segment lines points


Figure 22: Second discrete algorithm result

3. Discussion

I presented above two different kinds of analysis of conformal application problem: the boundary point association by using the Lagrange interpolation formula and the point pair association by going all over the given domains.

In the first case I studied different kinds of distributions of the given domain boundary points, but we can not say for neither distribution that is optimal. Probably so far the distribution that gives the best approximation for the optional given domain boundary is the uniform distribution. But in this case the received polynomial function with Lagrange interpolation formula isn't an injection.

The second method works only for starlike domains, and it's not sure to satisfy the conditions of definition 1.1 or definition 1.2.

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