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Discrete approximation*

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Abstract

In this paper we will approximate points which mark the geometrical position of special probes. The aim of the approximation is to generate a closed, compact and mushroom-shape surface which comply with the geological form of the methane. We will show a new technique which fulfils the geological requirement and which is more efficient than the known methods in the literature [1], [2], [3], [4], [5]. In this paper we will illustrate this comparison with expressive examples.

Categories and Subject Descriptors: I.3.5 [Computational Geometry and Object Modeling]: Curve, surface, solid, and object representations;

Key Words and Phrases: approximation-interpolation methods, geological reserve calculating

1. Introduction

The very spectacular development and the big opportunities of informatics brought about a radical change in the classical numerical methods. The continual and deterministic methods are going to be replaced by discrete ones that from a practical point of view work under a given limit of error. We are going to show through a given application such a new discrete numerical solution that approximates geometrical points in given conditions. The origin of the discussed problem is the modelling of real geological reserve calculating. This problem is about given n points that mark the geometrical position of special probes. (We have relatively small amount of points, because constructing a probe can be too costly). The data supplied by these probes constitutes the basis for drawing the geometrical shape of the methane.

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How should we begin graphically reconstructing the methane? Naturally first we look up in the literature, what the experts in the field propose. The literature proposes for drawing the surfaces by Gauss's less square method. To be exact he talks about polynomial and harmonic surfaces that are modeled with double Fourier series [1], [2], [3]. But neither of the methods deals with the logical requirement that the methane must have a compact form. One of the simple ways to examine this is that the contour lines of the surface have to be closed. It is a natural requirement that the methane must be compact, so finite and closed. How good it would be if some reserves of methane or petrol were infinite! So we have to look for another model. One of the new and strong devices of the modern numerical analysis is the spline. Each spline has a spatial and plan variant: Besier and B-Spline. Probably with the efficient use of some of these splines our problem can be adequately solved, but there are disadvantages. First of all the points have to be arranged in a grid to order for us to draw the surface. Secondly, it is true that the surface originated like this is in the convex hull of the grid, but this does not mean that our surface will be closed.

These arguments made us think of a new approximation, called discrete approximation which guarantees that we approximate the points with compact form. There is a philosophical argument too which supports our method. For us every piece of information is just the coordinate of points. Who can tell if the searched surface is polynomial or harmonic or just some special function? There is no argument supporting this choice. On the contrary, we are concerned about what we can achieve with points as only sources.

Furthermore it is an important geological requirement and an empirical fact that methane and petrol have the shape of a mushroom or a done. They cannot have a polyhedron like, plicate surface or any other rippling surface. These facts were justified by geological excavation. That is the reason why we could test only empirically the formula and solutions proposed by us.

2. Presentation of the method

Points $(x(i), y(i), z(i))$, $i = 1, \dots, n$ are given and we want to approximate these with a compact surface. The idea is the following: we look for a $z = F(x, y)$-like implicit function that we can analyze only in a discrete form, so point-like. We analyze them in the following form for given x and y values:

$$F(x,y) := (z_{max} - z_{min}) + \frac{\sum\limits_{i=1}^{n} d((x,y),(x(i),y(i)))z(i)}{\sum\limits_{i=1}^{n} d((x,y),(x(i),y(i)))}$$

$$+\frac{d((x,y),(x(k),y(k)))z(k)}{\text{factor}} + \frac{d((x,y),(x(l),y(l)))z(l)}{\text{factor}}$$

$$+\frac{d((x,y),(x(m),y(m)))z(m)}{\text{factor}}$$

where $d(A, B)$ is the Euclidian distance between points A and B, the factor is a constant real number with which we can increase or diminish the effect of the nearest 3 points. Its numerical value depends on the value of z (We chose it the following way: if $z(i)$ has two or three digits than factor $=$ 10.000, if it has 4 digits then factor $=$ 100.000. So we can decide the value of the factor only empirically. z_{max} , respectively z_{min} are the highest and the lowest z values. The k, l, m index values mark the 3 nearest points.

The

$$Fd(x,y) = \frac{\sum\limits_{i=1}^{n} d_i z_i}{\sum\limits_{i=1}^{n} d_i}$$

formula gives the essence of the approximation. Here we have to make an important remark. During the accomplishment of our research plan, and the examination of the literature we found out that there was already a very similar method to our own, an approximation-interpolation method [4], [5]. This is called the calculating of the arithmetic mean powered by the inverse of distance. This proved that we were on the right path and in an important field. The formula is:

$$Fi(x,y) = \frac{\sum\limits_{i=1}^{n} \frac{z_i}{d_i}}{\sum\limits_{i=1}^{n} \frac{1}{d_i}}.$$

Of course we can go on refining our method and we can take the square of the distances:

$$Fd^2(x,y) = \frac{\sum\limits_{i=1}^{n} d_i^2 z_i}{\sum\limits_{i=1}^{n} d_i^2}$$

the calculating of arithmetic mean powered by the square of the distance or more simply the arithmetic mean powered by distance.

Probably this is a mathematically not too significant formula, because it expresses a simple relation between a powered mean and some effect of the surrounding points. But let us take a look at the drawings. We get extremely beautifully surfaces with this formula. We had the contour lines and the spatial axonometrycal image of the surface drawn.

For admitting that the contour lines will be closed it is enough to admit that the contour lines will occupy the geometrical place of points that have constant potential energy. More exactly its distance measured from the three nearest points plus a powered mean are constant.

We have to admit that this surface is always compact. It is logical that the points $(x, y) \in [a, b] \times [c, d]$ can change in a closed rectangle-shaped region. In this case $F(x, y)$ cannot be highest than $2z_{max}$ and smaller than z_{min}.

Let us analyse the drawings. The first drawing was made with the Fd formula, the second with Fd^2, the third with Fi, the fourth with Gauss method, while the fifth with Lagrange's method. It is obvious that from the point of view of the primary requirement (compact and mushroom-shaped) the worst is the Fi formula, after that the Lagrange method while the Fd^2 gives a good solution.

In the first three cases we had the axonometrycal image together with the contour lines drawn. The drawings below were made on the basis of different data series and methods. With the last two methods we used the functions of the Maple and Matlab programs.

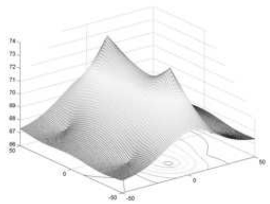
3. Conclusions

We called our method to be worked out discrete approximation. After the examination of the literature we found it more convenient to name the formulas in a more concrete way, as it follows: the arithmetic mean powered by distance, the arithmetic mean powered by the square of distance and the arithmetic mean powered by the inverse of distance. Of course all these kinds of methods can be called discrete approximation. So the discrete approximation could function as a collective concept.

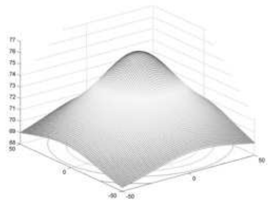
There are very small differences in the formulas, but from the point of view of the practical application they have a huge importance: the arithmetic mean powered by square of distance that we proposed meets almost entirely the system of requirements that we formulated: the surface is always compact and it has the shape of a mushroom.

As this is a hypothetical model, in the next step we should find an excavation site with real data to compare the excavated volume with the volume that we well receive with our method.

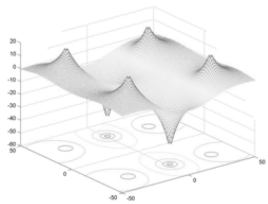
Data series 1
$X = [-31, -30, 1, 2, 30, 31];$
$Y = [30, -30, 31, -29, 29, -31];$
$Z = [20, 20, -50, -50, 20, 20];$



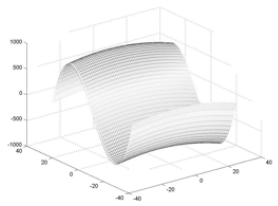
Fd_1



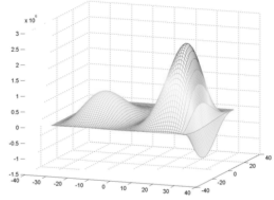
Fd_1^2



Fi_1



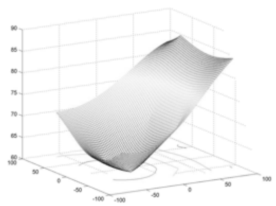
$Gauss_1$



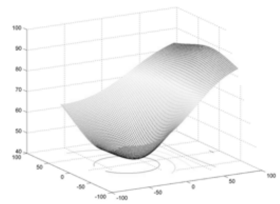
$Lagrange_1$

Data series 2

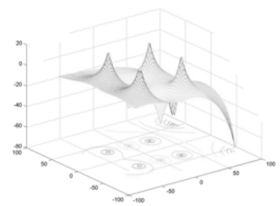
$X = [-31, -30, 1, 2, 30, 31, 90, 91]$;
$Y = [30, -30, 31, -29, 29, -32, 60, -60]$;
$Z = [20, 20, -50, -50, 20, 20, -80, -80]$;



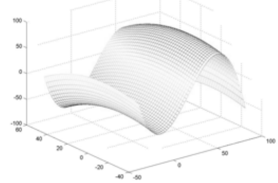
Fd_2



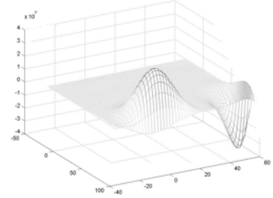
Fd_2^2



Fi_2



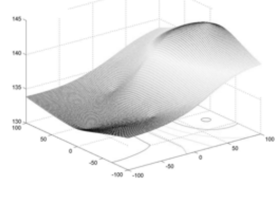
$Gauss_2$



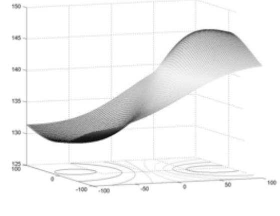
$Lagrange_2$

Data series 3

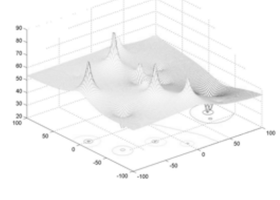
$X = [10, 39, 80, 20, 56, 77, 29, 18, 98, -25, -12, 68, 50, 30, -54, -86, 2, -7, 45, -70]$;
$Y = [20, 12, -23, 0, 41, -25, 35, 30, 100, -50, -16, -41, 77, 85, 61, 1, -79, 49, -85, -44]$;
$Z = [10, 30, 25, 47, 39, 10, 39, 70, 52, 89, 64, 23, 51, 68, 53, 89, 77, 96, 46, 30]$;



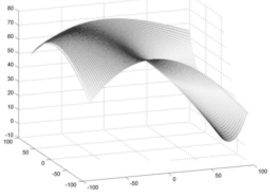
Fd_3

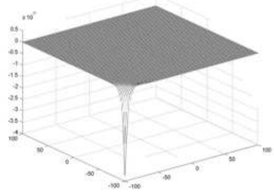


Fd_3^2



Fi_3

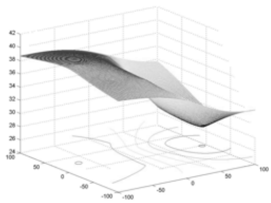


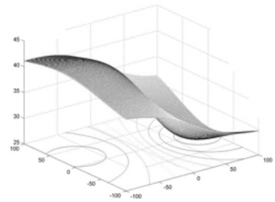


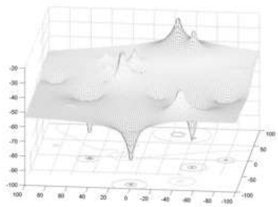
$Gauss_3$ $Lagrange_3$

Data series 4

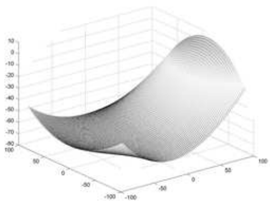
$X = [10, 39, 80, 20, 56, 77, 29, 18, 98, -25, -12, 68, 50, 30, -54, -86, 2, -7, 45, -70];$
$Y = [20, 12, -23, 0, 41, -25, 35, 30, 100, -50, -16, -41, 77, 85, 61, 1, -79, 49, -85, -44];$
$Z = [10, 30, 25, 47, 39, 10, 39, 70, 52, 89, 64, 23, 51, 68, 53, 89, 77, 96, 46, 30] * (-1);$

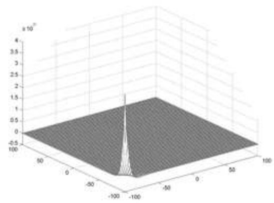






Fd_4 Fd_4^2 Fi_4

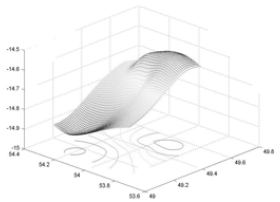




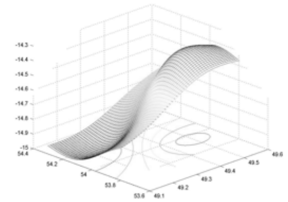
$Gauss_4$ $Lagrange_4$

Data series 5

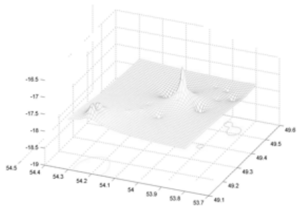
$X = [495492.00, 493859.00, 494593.00, 493601.64, 493329.50, 492790.30, 492345.99,$
$492660.71, 495104.87, 491737.32, 494365.64, 494743.82, 494116.98, 495493.50, 494780.18,$
$491268.56, 490744.75, 493878.82];$
$Y = [539437.00, 539120.00, 540812.00, 539894.23, 540970.67, 540217.93, 541224.26,$
$540916.71, 538445.50, 542150.12, 540361.50, 538880.43, 539803.83, 538917.87, 540406.97,$
$542143.40, 542628.66, 539856.97];$
$Z = [-1802.00, -1849.00, -1886.00, -1766.00, -1782.00, -1804.00, -1755.00, -1761.00,$
$-1830.00, -1706.00, -1801.00, -1803.00, -1749.00, -1812.00, -1856.00, -1684.00,$
$-1753.00, -1583.00];$



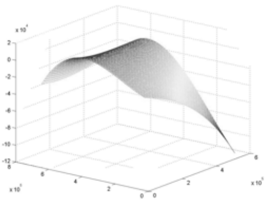
Fd_5



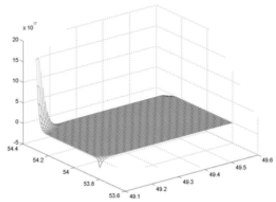
Fd_5^2



Fi_5



$Gauss_5$



$Lagrange_5$

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