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Possible Uses of the Disjunction Operator M

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Abstract

We discuss a combinatorial interdependence between the maximal number of nonredundant symbols on n objects, if these elements of a set are looked at as a commutative and – at the same time – as a sequenced assembly. We generate combinations of partitions on a set (= "structures on a set"; = "multidimensional partitions") and compare the maximal number of distinct structures on a set (= "n?") to the maximal number of distinct sequences that the elements of the set can be ordered into (= "$n!$"). We find that $\max(n?, n!) = f(n)$. Switching between the longitudinal and the transversal way of storing information appears to allow inroads towards understanding some puzzling phenomena in a wide variety of fields.

In the present paper, we concentrate on an important combinatorial detail which allows classifying (categorising, grouping) sets according to the maximal extent of inner disjunction of their subsets. We renumber the set N into a System M. We are still with additions of natural numbers (partitions), but select some, for which the translation function $m(n)$ yields also.t. For these, $m(n) = \sum m(n_i)$ above and beside $n = \sum n_i$. We look into possible uses of.d. (doubly /or more/.t.) sentences on N.

Categories and Subject Descriptors: Combinatorics, number theory, disjunction operator, symmetry definitions, size-invariant translations.

Key Words and Phrases: truth level.i. > 1; dissimilarity of parts of the whole, natural constants, extent of possibly being otherwise

1. Introduction

We treat a natural number n (an element of N) as a logical sentence stating about a set

a) that it has cardinality n;

b) that it is in a partitional state $E(n, 1)$[1], that is, that it is in one piece. The natural numbers appear thus as a special case – the maximally de-fragmented case – of a set. The set can be in any of $E(n)$ distinct onedimensional fragmentational states.

Each partition is of course.t., as it is of the form $n = \sum n_i$.

We call a sequence $1, 2, 3, \ldots i$ an N-enumeration as it fulfills following requirements:

1) each element is present $1 \ldots i$ and

2)no element from the sequence $1 \ldots i$ is present more than once, as is the case with N.

2. The M-translation

We set the natural numbers n=i*(i+1)/2 to 0 and fill out the places to the next 0 value with 0±k until the following picture appears:

N	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
M	0	-1	0	1	-1	0	1	-2	-1	0	1	2	-2	-1	0	1	2	-3	-2	-1

The rule to create this m-translation is as follows:

```
function m(n);
      local integer tmp, first, len, zero, m;
      tmp = int(sqrt(n/2));
      first = 2*tmp**2;
      len = 4*tmp + 2;
      zero = if(n-first \texttt{<} len/2, first + tmp, first + len/2 + tmp);
      m = n - zero;
return(m).
```

Each element of N has one value on M assigned to it. Each element of M can stand for an infinite number of elements of N.

We shall now discuss possible benefits coming from using the M-expressions of additions together with the usual picture on N.

3. Properties of the.d. sentences

3.1. Definition of.d. sentences

The set of.d. sentences is generated by selection from among:

all partitions of a natural number in the form of $n = \sum n_i$.

those partitions for which it is true that $m(n) = \sum m(n_i)$.

E.g. 3+3=6 is.d. because 0m + 0m = 0m, as is 4+2=6 or 1+1+1+1+1+1=6, but 2+2+2=6 is not.d., although.t., because -1m + -1m + -1m # 0m

[1] $E(n)$: the number of partitions of n; $E(n, i)$: the number of partitions of n into i summands

3.2. Absolute frequency of.d. sentences

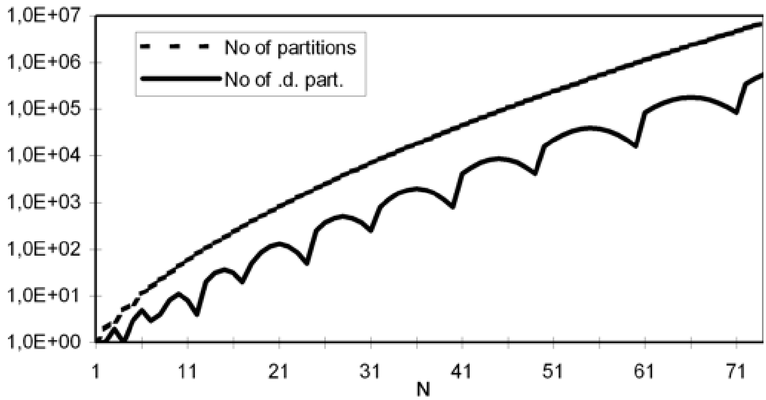

Figure 1: Absolute number of.d. additions on N

We see that there is a symmetry in absolute numbers centered on m0 values (3, 6, 10, 15, ... etc. on N). As $E(n_i) > E(n_j)$ for any $i > j$, the proportion.d./.t. will show a different picture.

3.3. Relative frequency of.d. sentences

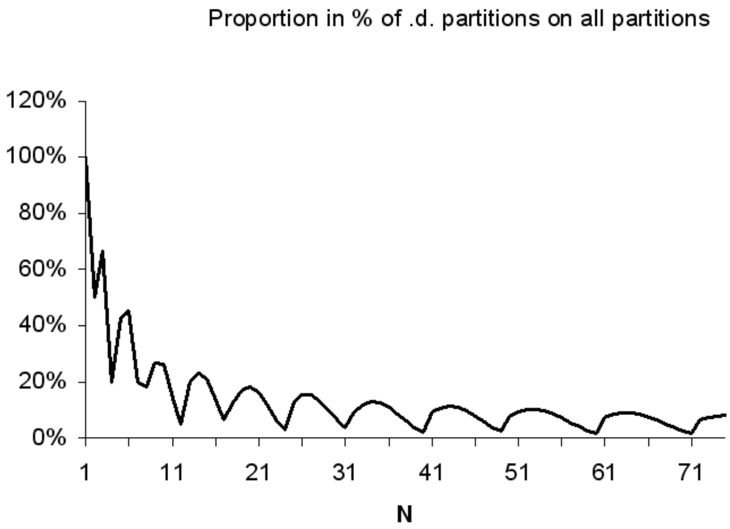

Figure 2: Relative number of.d. additions on N

It appears that – roughly, as a statistical trend – about 10 % of all logical sentences on N have the property of being.t. in an additional sense.

3.4. Most probable fragmentational state of a set

As each partition of a natural number is exclusive and the collection of partitions is exhaustive, we may treat the collection of partitions of n into k summands as a probability density. Let us name the distribution of partitions of a number n into k summands the Euler distribution. It shows a stark contrast to the Gauss distribution, most strikingly by its assymmetric nature.

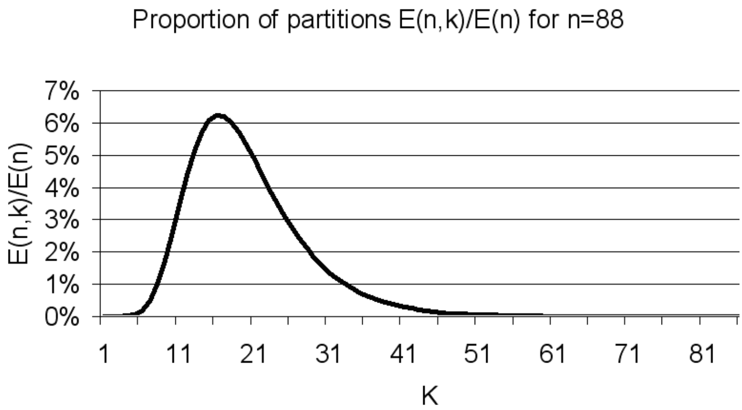

Figure 3: Probability for a set to consist of k subsets

As we have the overall size of the set and the post probable number of subsets, we also have the most probable size of a subset. The assymmetric peak at $k = k_{\max}$ tends to include almost all cases as n grows. The relation of $k_{\max}$ to n appears to be closely linked to the *sqrt()* function[2].

3.5. Most probable distribution of.d. subsets

Combining the M and the Euler distributions one can conclude the probability densities for a most typical subset consisting of.d. sentences. This Figure cannot be reproduced here, as it is basically 4-dimensional.

We have seen that the absolute number of.d. sentences is symmetric to the next m0 value on N. Therefore, there is no monotone increase of.d. sentences as n grows.

There is a preferred range of k (called $k_{\max}$) for each n: $k_{\max}$ grows much slower than n.

The frequency of.d. sentences is then $f(n, k, m)$.

[2]This observation is from Prof. Gerd Baron of the TU Wien

4. Possible applications of.d. sentences

4.1. A formal approach to symmetry

The M-translation allows disussing the idea and concepts of symmetry as such. The idea of symmetry as such is a concept that has eluded formalisation in a stubborn fashion. We may now offer a definition of two kinds of symmetry: the axe and the mirror symmetry.

The axe symmetry has an axe that is not repeated on the other side of the symmetry. The axe constitutes the symmetry, relative to which a distance is built up. In the example with 3, the element 0 is the axe in the symmetrical sequence –3,-2,-1,0,1,2,3. The axe symmetry is in this example 7 long.

Example for 3:

N	18	19	20	21	22	23	24
M	-3	-2	-1	0	1	2	3
	a	*a*	*a*		*b*	*b*	*b*

The elements denoted *a* are symmetric to the elements denoted *b*. There exists a central, neutral element.
The mirror symmetry has no central, zero element, and may be pointed out in the inner, bigger repetition. See e.g. for 3: -3,-2,-1,0,1,2,3 is the first half of the repetition, the second half is identical, and there is no neutral element between the repetitions. (There is no element in the symmetry that is not twice.) The mirror symmetry is in this example 14 long.

Example for 3:

N	18	19	20	21	22	23	24	25	26	27	28	29	30	31
M	-3	-2	-1	0	1	2	3	-3	-2	-1	0	1	2	3
	a	*a*	*a*	*a*	*a*	*a*	*a*	*b*	*b*	*b*	*b*	*b*	*b*	*b*

The elements denoted *a* are symmetric to the elements denoted *b*. There exists no central, neutral element.

Both the axe and the mirror symmetry match elements with identical value but differing polarity sign data.

4.2. Information about the disjunction of a set without regard to its size

We envision a situation, where we are less concerned about the *absolute size* of an assembly, but rather about its *inner homogeneity.* By transmitting the m-translation, we know, how close the subsets are to an N-enumeration (whether each element's size is present exactly once). This information allows including or excluding classes of fragmentational states of a set from a search. Of course, one looses the absolute size property, but one has the number of fragments, therefore there is a most probable stretch on N where this would fit well.

We may not know – by using this procedure – how big an assembly is, but we know how uniform resp. varied its parts are. Having an information about how many parts something is built up, and how differing these parts are among each other, does allow – by probabilistic means – to conclude, which of the possible sizes we may most probably have to look for.

4.3. The left and right parts of the mirror symmetry

The two halves of a period stretch on N between $2i^2$ and $2(i+1)^2-1$. Left and right parts are identical on M but differ on N.
We state – without being able to give any numeric proof of this assumption – that the left and right parts of a period can serve as conceptual pictures for uniform building blocks of material. The reasoning behind the assumption is as follows:

We discuss objects (elements of a set) and their logical relations among each other. We match the number of objects and the number of logical relations the objects can carry to each other. E.g., we know that we can represent up to $n!$ distinct logical relations on n objects if we treat the objects as a sequential collection. We have shown that we can represent up to $n?$ distinct logical relations on n objects if we read these relations off the assembly as a contemporal (nonsequenced) collection (see [1]). These are the upper limits of message carrying capacities of objects.

If we have the maximal number of logical relations per n objects, then we have also the minimal fraction of an object per logical relation (under the assumption that we use n objects).

We now assume that *congruent* logical relations can be distinguished from logical relations that are not congruent. The congruence may be visualised as the linearisation of a group structure (the "rolling down" of a mixture into a sequence), where the group boundaries in the contemporal assembly do not hinder the linear neighbourhood relations. (E.g. there is no difficulty linearising a set, of which the elements are either in the $E(n,1)$ or in the $E(n,n)$ partitional state.) Another approach is the usage of the m-translation.

The basic duality (dichotomy, symmetry) of the model reappears here again. We state that logical relations that are congruent are a subset of logical relations, therefore cannot be more than these. Thus, the $\min(n!, n?)$ will give the upper limit of congruent logical relations that can be there on a set of n objects. Looking into the relation between n, $n!$ and $n?$, one observes (see [2]) that

a) there is a slack around $n?/n!_{135}$ that allows the translation of the density of logical relations into the existence of a carrier object, and

b) near $n = 140$, the quotient $n?/n!$ will approach 0. We interpret this as showing a probability near zero for a congruence of logical relations among all relations if the size of the assembly is above that number.

We therefore have a conceptually *finite range on N* whereon an interpretation of an m-value on N can make sense.

Having now a finite collection of logical relations, where each logical relation can count as a specific portion of a ("physical", carrier) object, we have a finite number (of differing amounts) of fractions of an object. Then we have two density heaps on M – that are slightly different on N. The right half-period is in its expectation value higher than the left one.

Being identically dense with respect to the picture on M, and being slightly smaller resp. bigger with respect to the picture on N allows us to conceptualise two basic kinds of building blocks, of which one is slightly "more" than the other, but this additional fraction of an object cannot be within the object.

This concept does remind some of the idea "neutron" and "proton+electron".

4.4. The order of subsequences of the sequence M on N

The recoding of N into M yields a sequence which runs for $n\{1, 2, 3, \dots\}$ like $m\{0, -1, 0, \dots\}$. One observes that there are two instances of elements coming to lie next to each other that have differing signs: a) within a period, between the left and the right half-periods, and b) after the end of a period, at the beginning of the next period. In case a) the elements with differing signs have an identical numerical extent on M. In case b) the elements with differing signs have a differing numerical extent on M, where the right one has an identical absolute value to the successor on N to the last element of the left period. The elements are in both cases predecessor-successor on N.

That positive and negative come to lie next to each other, like in the case of our concept of magnetic phaenomena, has led to the naming of the system as M for magnetic.

The first element of each period on N has a value on N that agrees to the rule $n = 2i^2$. This is also the rule for the commencement of a new electron layer.

5. Discussion

This curious sequence opens up new ways of looking at numbers. Each number is seen as an assembly. The assembly has several properties to it, among which the size ("cardinality") is but one among others. We let loose of the "size" meaning of a number and look into its implicit meaning: a) made up of identical parts and b) is in one piece. (We learn that 6=1+1+1+1+1+1 at school.) We fragment the assembly into pieces and then count and classify the pieces we find.

We see that the overall size of an assembly and the number of chunks it is most probably in have a rather strict relationship. For reasons of space, we could not look more deeply into the most probable number of fragments (denoted $k_{\max}$) here. Here, we have discussed in an overview the other question: how uniform are the parts in reality? We have shown that there is a property of a number that describes the maximally possible disjunction among its summands. Measuring by means of N treats each different object to be measured – however differing in qualities they are – as having something in common, some property that can be matched to N, which is itself a collection of identical elements. (The basic concept of N is that it is made up of a lot of $1 - s$.) On M, we use a background made up of units of those each is distinct to the other. We assume a common property of all things to be measured on M, namely their property of being made up of distinct units (differing parts).

The congruence between descriptions, where once we state that several different assemblies consist of parts that are differing (or identical) before a background of identical elements (like we state about a chair and a table that both are similar in their properties of being x and y cm long, because we use a measuring tape made up of many identical small elements, each the same length), and once that they are alike in being made up of a similar proportion of distinct to identical sub-elements (like we compare a watch and a piece of coal on one hand and a small plant and a bucket of sand on the other) – this congruece of combing together complex and simple, big-and-simple and small-and-complex (and other combinations of the properties: has differing parts, is made up of identical parts), may turn out to help understanding how our concepts are organised.

References

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