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A computer-assisted proof for stable/unstable behaviour of periodic solutions for the forced damped pendulum

Balázs Bánhelyi and László Hatvani

University of Szeged
Institute of Informatics
e-mail: banhelyi@inf.u-szeged.hu

Abstract

We consider a simple mechanical system, the forced damped pendulum. According to earlier publications this system has chaotic behaviour. Among other things, the chaotic behavior is implied by the existence of an unstable periodic solution.

First we describe an automatic technique that is able to enclose periodic solutions. To find such regions that satisfy the respective conditions, we applied an interval inclusion and a recursive subdivision technique. In the considered system we find two periodic solutions. One of them is unstable and the other one is stable.

To find out whether the periodic solution is stable, we can calculate the Jacobian matrix for the related variables and analyze their eigenvalues. Unfortunately, the Jacobian matrix of this pendulum cannot be given in closed form. One of the possibilities is to apply the characteristic multiplicator technique. To this end we can determine the Poincaré map, and while we follow the trajectory, we calculate the derivatives as well. With this method we can provide the anticipated result for the mentioned 2π-periodic solutions.

Finally, we discuss some stabilization techniques. We analyze how to stabilize the unstable periodic solution of the considered pendulum. One of these methods uses the current values of the state variables (the angle and the speed of the pendulum), thus this is a feedback control. The other method does not use this kind of information, so it is not a feedback control. We also analyze whether these methods really stabilize the unstable solution. We are able to prove necessary conditions for the stabilization of the unstable solution.

Keywords: computer-assisted proof, periodic solutions, forced damped pendulum, control

Balázs Bánhelyi
6701 Szeged, Hungary, P.O. Box 652.