A FUNCTIONAL CENTRAL LIMIT THEOREM FOR KERNEL TYPE DENSITY ESTIMATORS

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In this paper a functional central limit theorem in the space $L_2[0,1]$ is proved for kernel type density estimators for α-mixing random fields if the locations of observations become more and more dense in an increasing sequence of domains.

In [1] the asymptotic normality of kernel type density estimators is proved for α-mixing sequences and continuous time processes. In [3] a so called infill-increasing setup is used to obtain a result that is in some sense between the discrete and the continuous time cases.

In statistics, most asymptotic results concern the increasing domain case, i.e. when the random process (or field) is observed in an increasing sequence of domains T_n, with $|T_n| \to \infty$. However, if we observe a random field in a fixed domain and intend to prove an asymptotic theorem when the observations become dense in that domain, we obtain the so called infill asymptotics (see [2]). In this paper we combine the infill and the increasing domain approaches. We call infill-increasing approach if our observations become more and more dense in an increasing sequence of domains. Using this setup, in [5] the asymptotic behaviour of the empirical distribution function is studied.

Let $\{\xi_{\mathbf{i}} \,|\, \mathbf{i} \in \mathcal{D}_n\}$ be the set of observations of a random field. Here $\mathcal{D}_n$ denotes the set of locations where the random field is observed. Let K be a kernel and let $h_n > 0$, then the kernel-type density estimator is

$$f_n(x) = \frac{1}{|\mathcal{D}_n|} \sum\nolimits_{\mathbf{i}\in\mathcal{D}_n} \frac{1}{h_n} K\left(\frac{x-\xi_{\mathbf{i}}}{h_n}\right), \qquad x \in \mathbb{R}. \tag{1}$$

In [3] the asymptotic normality of the kernel type density estimator (1) is proved in the infill-increasing case. The main result of this paper is a functional central limit theorem for $f_n(x)$. We prove our functional central limit theorem in $L_2[0,1]$, i.e. in the space of square integrable functions defined in the interval $[0,1]$. We apply criteria for functional limit theorems in L_p established in [4].

References

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