Local asymptotic normality of unstable HJM type interest rate model

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We consider a discrete time Heath-Jarrow-Morton (HJM) type forward interest rate model driven by a spatial autoregression process. Let $\{\eta_{i,j} : i, j \in \mathbb{Z}_+\}$ be i.i.d. standard normal random variables on a probability space $(\Omega, \mathcal{F}, P)$, where $\mathbb{Z}_+$ denotes the set of nonnegative integers. Let $\rho \in \mathbb{R}$ be the autoregression coefficient. Let $f_{k,\ell}$ denote the forward interest rate at time k with time to maturity date ℓ $(k, \ell \in \mathbb{Z}_+)$. We assume that the initial values $f_{0,\ell}$ are known at time 0. If this market is arbitrage free then the forward rates are given by the following equations (see [1]):

$$f_{k,\ell} = f_{k-1,\ell+1} + \rho(f_{k,\ell-1} - f_{k-1,\ell}) + \eta_{k,\ell} + \frac{1}{2}\sum_{i=0}^{2\ell} \rho^i, \qquad (k, \ell \in \mathbb{N}).$$

Theorem. Suppose that the true parameter is $\rho_0 = 1$. Let $\{K_n, L_n : n \in \mathbb{N}\}$ be positive integers such that $K_n = nK + o(n)$ and $L_n = nL + o(n)$ as $n \to \infty$ with some $K > 0$ and $L > 0$. For each $n \in \mathbb{N}$ let $\hat{\rho}_n$ denote a maximum likelihood estimator of ρ over $[A, 1]$ based on a sample $\{f_{k,\ell} : 1 \le k \le K_n,\, 0 \le \ell \le L_n\}$, where $A \in (-1, 1)$. Then $\hat{\rho}_n$ is strongly consistent, i.e.

$$\hat{\rho}_n \to \rho_0 \qquad \text{almost surely as } n \to \infty.$$

Moreover the sequence of the related statistical experiments is locally asymptotically normal (LAN) in the following sense:

$$\frac{\mathcal{L}_n\left(f_{k,\ell} : 1 \le k \le K_n,\, 0 \le \ell \le L_n; 1 - \frac{h}{n^3}\right)}{\mathcal{L}_n\left(f_{k,\ell} : 1 \le k \le K_n,\, 0 \le \ell \le L_n; 1\right)} \xrightarrow{\mathcal{D}} \exp\left\{h\zeta - \frac{1}{2}\sigma^2 h^2\right\}, \quad \forall h \in \mathbb{R}_+,$$

where $\mathcal{L}_n$ denotes the likelihood function of the sample, $\zeta \sim \mathcal{N}(0, \sigma^2)$ and

$$\sigma^2 = \frac{9}{4}\int_0^K \left(\int_0^L t^4 dt\right) ds + \frac{9}{4}\int_0^K \frac{1}{s}\left(\int_L^{L+s} t^2 dt\right)^2 ds.$$

References

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