

A parallelized computer-assisted proof of chaos ^{*}

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We report on experiences with an adaptive subdivision method [1] supported by interval arithmetic and parallel techniques. Our parallel computing system is implemented for a computer, which has four processors and shared memory. The Branch and Bound technique shares the generated subintervals between the processor, hence the calculation is became a faster by more processor.

This method enables us to prove subset relations of the form $\mathcal{T}(W) \subset U$ and thus to check certain sufficient conditions for chaotic behavior of dynamical systems in a rigorous way. The novelty is that the process of gaining the subset relations to be checked is, to a large extent, also automatized. The computational methods are demonstrated by finding planar subsets with chaotic behavior for the forced damped pendulum [3], and a Hnon map [1, 2]. This two dynamical systems need a large amount of calculation, so the parallelized technique is very useful. We give details on the parallelization techniques applied.

References

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