

On Low Degree Optimal Lagrange Interpolation via Symbolic Computation

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Abstract

We report on optimal Lagrange interpolation of degree at most five on the unit interval $[-1, 1]$. We start with the case of optimal quadratic interpolation: In a largely unknown paper, Schurer (1974, Stud. Sci. Math. Hung. 9, 77–79) has analytically described the infinitely many zero-symmetric and zero-asymmetric extremal node systems $-1 \leq x_1 < x_2 < x_3 \leq 1$ which all lead to the minimal Lebesgue constant $5/4$ that had already been determined by Bernstein (1931, Izv. Akad. Nauk SSSR 7, 1025-1050). First, we reconstruct Schurer's result via symbolic computation. Secondly, we also provide some new analytical results on optimal Lebesgue constants and extremal node systems via symbolic computation if the degree is greater than two. This is a joint work with H.-J. Rack and the continuation of the work recently presented by Rack (Springer Proceedings in Mathematics and Statistics 41, 117-120, 2013) and by Vajda (SCSS 2013, EPiC Series, vol. 123).

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