Computer visualization of higher dimensional and non-Euclidean geometries

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Abstract

The theoretical background of our topic is the d-dimensional projective spherical space $\mathcal{PS}^d$ $(\mathbf{V}^{d+1}; \boldsymbol{V}_{d+1}; \mathbf{R}; \sim)$ or projective space $\mathcal{P}^d$, modelled as subspace incidence structure of the real $d+1$-dimensional vector space $\mathbf{V}^{d+1}$ for points or its dual $\boldsymbol{V}_{d+1}$ for hyperplanes, respectively. Here $\sim$ indicates the multiplicative equivalence by positive reals $\mathbf{R}^+$ in case of $\mathcal{PS}^d$, or by non-zeros $\mathbf{R} \setminus \{0\}$ for $\mathcal{P}^d$. E.g. non-zero $\mathbf{V}^{d+1}$ vectors $\mathbf{x} \sim c\mathbf{x}$ describe the same point $X(\mathbf{x})$ in $\mathcal{PS}^d$ iff $c \in \mathbf{R}^+$.

In this presentation we report some new results in visualizing higher dimensional regular polytopes in Euclidean 4-space $\mathbf{E}^4$ (see e.g. [1] and [2]). Furthermore, we illustrate in $\mathbf{E}^3$ some new analogues of the classical objects: polyhedra, spheres, balls, their combinations in dense ball packing problems in the so-called Thurston 3-geometries $\mathbf{E}^3$, $\mathbf{S}^3$, $\mathbf{H}^3$, $\mathbf{S}^2 \times \mathbf{R}$, $\mathbf{H}^2 \times \mathbf{R}$, $\widetilde{\mathbf{SL_2R}}$, **Nil**, and **Sol** (see e.g. [3]). To these last illustrations ve have to specify the scalar product $<;>$ in $\boldsymbol{V}_{d+1}$ so in $\mathbf{V}^{d+1}$ $(d=3)$ by the signature and other requirements (as in [4]).

Keywords: Computer Aided Geometric Modeling, Computer Graphics, Computer visualization

References

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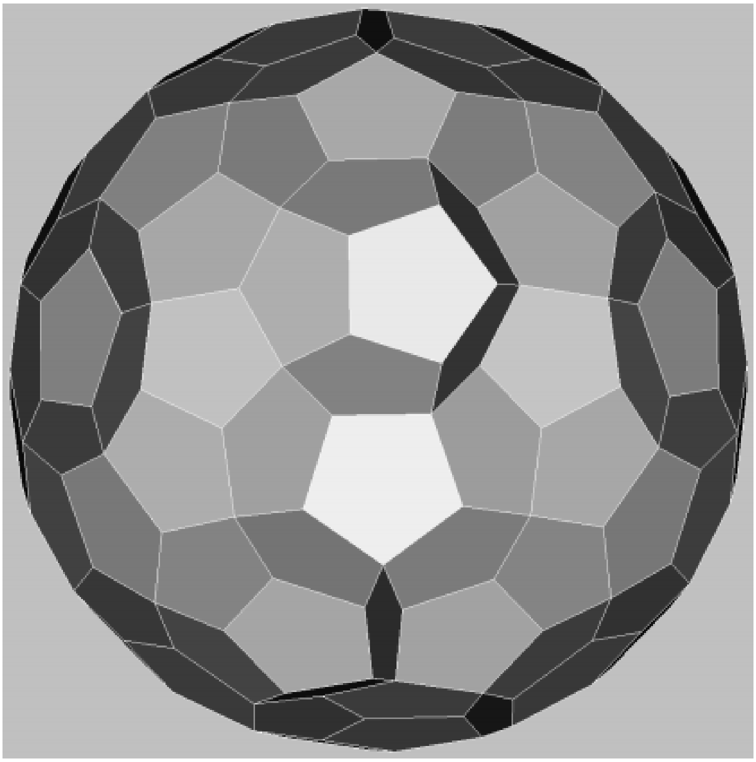

Figure 1: The 120-cell with Coxeter-Schläfli symbol (5, 3, 3)