Helical Structure of Space-filling Mosaics Based on 3D Models of the 5D and 6D Cubes

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Abstract

The 3-dimensional framework (3-model) of any *k*-dimensional cube (*k*-cube) can be produced either based on starting k edges arranged by rotational symmetry or as sequences of strut-chains originated from a separate one whose breakpoints join a single helix. Increasing the number of segments in the strut-chains to large *k* = n (infinity), continuous helices are created, whose Minkowski sum can be called n-zonotope [1-3]. Combining $2<j<k$ edges, we can build 3-models of *j*-cubes, as parts of a *k*-cube. The suitable combinations of these zonotope models can result in 3-dimensional space-filling mosaics [2-4]. The investigated periodical tessellations hold the 3-model of the *k*-cube and necessary *j*-cubes derived from it and follow the helical structure of our models. Such a mosaic can have fractal structure as well, since we can replace it with a restructured one, built from multiplied solids. These are composed by addition of 3-models of *k*- and *j*-cubes and are similar to the original ones. The intersections of the mosaics with planes allow unlimited possibilities to produce periodical symmetric plane-tiling. Moving intersection planes result in series of tessellations transforming into each other. Planar and spatial symmetry groups are the base of several works in different branches of art. The newer results can hopefully aid the correspondence among geometry, art and design [3-4].

Keywords: constructive geometry, hypercube modeling, tessellation, fractal, design

MSC: 52B10; 52B12, 52B15, 65D17

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